ID Number M

Entrance Examination for Master's Course, 2013 April Enrollment

Department of Quantum Matter

Department of Semiconductor Electronics and Integration Science

Graduate School of Advanced Sciences of Matter

Hiroshima University

MATHEMATICS

August 27, 2012, 10:30～12:00

Notices

(1) This booklet includes the following sheets.

Question sheets (including this sheet)	2 pages
Answer sheets	3 pages
Memo sheet	1 page

(2) There are three problems, [1]～[3].

(3) One answer sheet should be used for one problem. Write the problem number at the left upper corner of each answer sheet. The backside of the sheets can be used.

(4) Write your identification number on all the sheets.

(5) Return all the sheets listed in (1) at the end of the test.

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MATHEMATICS

[1] Answer the following questions on the differential equations.

(1) For the differential equation $y'' - 3y' + 2y = 0$, find the general solution and the particular solution that satisfies the initial condition $y(0) = 0$ and $y'(0) = 1$.

(2) For the differential equation $xy' + y = 2x$, find the general solution and the particular solution that satisfies the initial condition $y(1) = 2$, where $x \neq 0$.

[2]

(1) Find all eigenvalues and eigenvectors of the 2×2 matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

(2) Show the procedure to diagonalize the $N \times N$ matrix A, when all eigenvalues λ_j and corresponding eigenvectors u_j are given $(j = 1, 2, \cdots, N)$, where the family of eigenvectors is linearly indepedent.

(3) An $N \times N$ matrix A is Hermitian $(A^\dagger = A)$, where $A^\dagger = (A^t)^*$ and A^t denotes the transposed matrix of A.

(i) State and prove the property or properties of the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_N$.

(ii) State and prove the property or properties of the set of the eigenvectors $\{u_1, u_2, \cdots, u_N\}$, where the eigenvalues are all different.

[3] Answer the following questions.

(1) Suppose the curved surface given by $\phi(\mathbf{r}) = x^2 + y^2 - z^4 = C$ in the rectangular coordinate system O-xyz, where $\mathbf{r} = (x, y, z)$ and C denotes a nonzero constant. Find all unit vectors $\mathbf{n}(\mathbf{r})$ perpendicular to the surface at $\mathbf{r}$.

(2) Evaluate the definite integral

$$I = \int_{-\infty}^{\infty} \mathrm{d}x \, \mathrm{e}^{-ax^2},$$

where a denotes a positive constant.

(3) Illustrate the polar coordinates (r, θ, φ) in the rectangular coordinate system O-xyz drawing a figure, and express x, y and z by equations in terms of r, θ and φ.

(4) Find the volume V of the parallelepiped with the edges $\overline{\text{OA}}$, $\overline{\text{OB}}$, and $\overline{\text{OC}}$, in the rectangular coordinate system O-xyz, given the points A:$(1, 1, 0)$, B:$(2, 4, 5)$, and C:$(3, 0, 1)$.