

Entrance Examination

Department

Graduate School

(Engineering)

Answer Sheet

- (1) This booklet includes the
Problem sheets (including
Answer sheets
Memo sheet
- (2) There are five problems
boxes .
- (3) Solve three problems out of five.
- (4) One answer sheet should be used for each problem
and its category name and category number should be written on
The backside of the sheet.
- (5) Write your identification number on the backside of the sheet.
- (6) Return all received materials to the examination room.
- (7) Write the numbers of the problems you solved on the backside of the sheet.

[1] Electromagnetism

1. Consider a solid conducting sphere A with radius a and a hollow conducting sphere B with inner radius b (Fig. 1). The center of A is fixed at a positive value and that of B is fixed at a negative value. The dielectric constant of vacuum is ϵ_0 . r is the radial distance from the center of A in the spherical coordinate system.

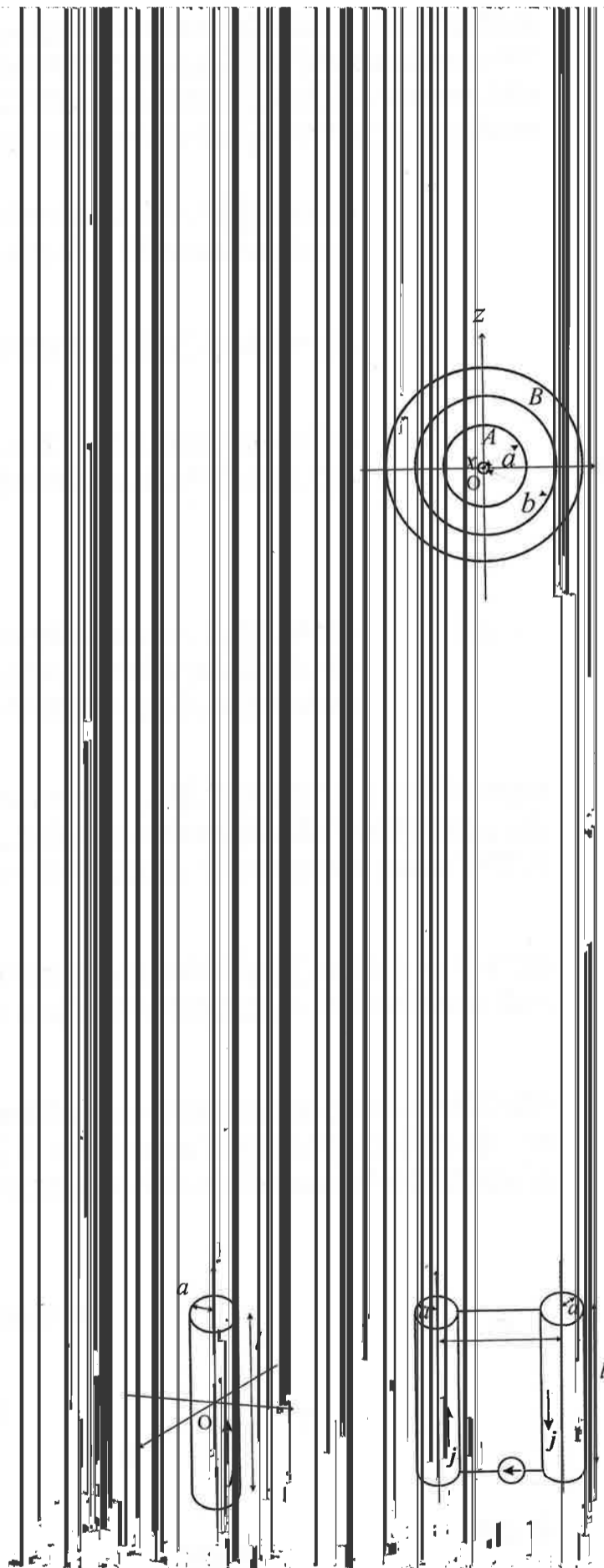
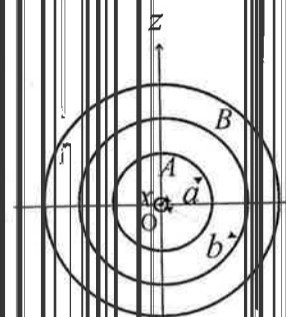
- (1) Give the direction and the magnitude of the electric field $E(r)$ at a point between A and B . The surface charge density on the surface of A is σ .
- (2) Draw a graph of the magnitude of the electric field $E(r)$ as a function of r .
- (3) Give σ , when the electric potential of the electric field $E(r)$ and the electric potential $V(r)$ at a specific value of V_1 .
- (4) Give the capacitance C for A and B .
- (5) Show that the electric potential $V(r)$ satisfies Laplace's equation. Since the system is spherically symmetric, the Laplacian in the spherical polar coordinate system is given by $\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r})$.

2. Consider a cylindrical conductor placed on the z -axis with length l . A current with a current density $\mathbf{j} = j \hat{z}$ flows in the positive z -direction. The conductor is sufficiently long and the effect of both end faces is neglected. The permeability of vacuum is μ_0 .

- (1) Give the magnitudes and the direction of the magnetic field $H_o(r)$ outside the conductor and the field $H_i(r)$ inside the conductor along the z -axis.

Another conductor having the same diameter is placed parallel to the first one to make a circuit as shown in Fig. 2. The magnetic field inside of the conductors and the displacement current is neglected.

- (2) Give the magnetic flux Φ through the surface S consisting of these conductors.
- (3) Give the self inductance L of the circuit.



Entrance Examination for April 2012 Enrollment
Graduate School of Advanced Sciences of Matter, Hiroshima University
Department of Quantum Matter (Engineering Field)

[2] Quantum Mechanics

Consider the ground state of a particle of mass m in a potential $V(x)$ that is infinite at $x < 0$, zero for $0 < x < L$, and a constant value of U for $x > L$,

$$V(x) = \begin{cases} +\infty & \text{for } x < 0 \\ 0 & \text{for } 0 < x < L \\ U & \text{for } x > L \end{cases}$$

The ground state energy E has a value between zero and the potential U .

1. Write down the time independent Schrödinger equation for $\psi(x)$ with energy eigenvalue E .
2. Give the boundary condition at $x = 0$ and $x = L$.
3. Find a wavefunction $\psi_1(x)$ that satisfies the boundary conditions at $x = 0$ and $x = L$ and satisfies the Schrödinger equation for $E < U$.
4. Give the boundary condition for $\psi(x)$ at $x = L$ for $E > U$.
5. Find a wavefunction $\psi_2(x)$ that satisfies the boundary conditions at $x = 0$ and $x = L$ and satisfies the Schrödinger equation for $E > U$.
6. Give the boundary conditions for $\psi(x)$ at $x = L$ for $E < U$.
7. For a specific value of $U = U_0$ find the ground state energy E at $E = U_0/2$. Use the boundary conditions at $x = L$ for $E < U$ and $E > U$ is given by

0

[3] Semiconductor E

Consider the drift motion of electrons in a semiconductor. The equation of motion for the drift velocity $v(t)$ at time t is

$$\frac{dv(t)}{dt} = \frac{eF}{m^*} - \frac{v(t)}{\tau} \quad (\text{a})$$

Here, e and m^* are the elementary charge and the effective electron mass, respectively, and τ is a constant. Let F be a non-zero constant electric field.

1. Find $v(t)$ for $t \geq 0$ by solving equation (a).
2. For large t , $v(t)$ approaches an asymptotic value. Find this value.
3. Draw a graph of $v(t)$ for $t \geq 0$. In the graph, indicate the asymptotic value.
4. Express the electron mobility μ using the asymptotic value.
5. Give the relation between the electric field F and the drift velocity v with an electron density of n .
6. Explain the physical meaning of the asymptotic value.

Quantum mechanically, the wave nature of electrons must be considered. Let us assume the following relation between the wavenumber k and the eigenenergy E of electrons;

$$E = E_0 + \frac{\hbar^2}{2m^*}(k - k_0)^2 \quad (\text{b}).$$

Here, $\hbar = h/(2\pi)$ with h being the Planck constant and E_0 and k_0 are a constant of energy and a constant of wavenumber, respectively.

7. Give the relation between E and the wavenumber k .
8. The particle-like motion described by the wavepacket of the electron. In the wavepacket, the central wavenumber k_C is given by $v = d\omega/dk$, where ω is the angular frequency and the wavenumber k . Derive the relation between the central wavenumber k_C and the energy E . Draw a graph of v as a function of k_C .
9. Using the equation $dk_C/dt = G/\hbar$ for an external force G , derive the first term of the expansion of v for small G .

[4] Thermodynamics

1. Explain briefly the canonical distribution and give numerical expressions for them. Use all the symbols used.
Keywords: thermal bath, probability, ϵ

2. Consider a system of N atoms in a line. State 2, whose energies are ϵ_1 and ϵ_2 respectively, where $\epsilon > 0$. We assume $\epsilon_1 < \epsilon_2$.

- (1) Find the number of cases in which N_1 atoms are in state 1 and N_2 atoms are in state 2.
- (2) For the case described in (1), give the value of the entropy S . Assume $N \gg 1$, $N_1 \gg 1$ and $N_2 \gg 1$.

$$\ln n!$$

3. Consider the case when the system is in contact with a thermal bath of absolute temperature T .

- (1) Find the number N_1 of atoms in state 1.
- (2) Find the values of N_1 and the entropy S in the high temperature limit $T \rightarrow \infty$.
- (3) Explain the results obtained in (2) and the relation between the Helmholtz free energy A and the entropy S .

**Entrance Exam
Graduate School of Advanc**

[5] Materials Science

1 . Solve the following questions concerning adsorption on a solid surface.

(1) Write down the rate equation for adsorption and desorption are k_a

(2) Write down the equilibrium condition. Here, K is

(3) The table shows data for a gas adsorption on a surface at a constant temperature. The data satisfy the Langmuir isotherm. which was defined in question (2). The volume $V(p)$ gives the value

2 . Answer the following questions concerning electrochemistry.

(1) Give the standard cell potential for the reaction. ΔG^0 , and the Faraday

(2) Give the logarithmic value of the equilibrium constant at a temperature of 300K. Here, F is considered to be $F = 1 \times 10^5 [C]$.

(3) Explain the nature of the equilibrium.

3 . Answer the following questions concerning X-ray diffraction techniques.

(1) As shown in the figure, a diffraction spot is observed at the angle θ which corresponds to the diffraction plane of a cubic crystal. Express the lattice constant a using λ and θ

(2) The C-H bond length of Sucrose was determined to be 0.110 nm by X-ray diffraction. What was obtained in these techniques?