ID Number	M

Entrance Examination for Master's Course, 2012 April Enrollment

Department of Quantum Matter

Department of Semiconductor Electronics and Integration Science

Graduate School of Advanced Sciences of Matter

Hiroshima University

MATHEMATICS

August 22, 2011, 10:30～12:00

Notices

(1) This booklet includes the following sheets.

Question sheets (including this sheet)	2 pages
Answer sheets	3 pages
Memo sheet	1 page

[redacted]

(3) One answer sheet should be used for one problem. Write the problem number

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MATHEMATICS

[1] Answer the following questions on the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x-y+3}{x-y}.$$

(1) Find the general solution.

(2) Find the solution which satisfies the conditions $x = 0$ and $y = 1$.

(3) Draw the solution of (2) on the xy plane.

[2] Answer the following questions.

(1) Suppose that any two-dimensional real vector $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is expressed as

$$\vec{x} = c_a\vec{a} + c_b\vec{b} \qquad ①$$

in terms of two vectors

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

with an appropriate set of scalar constants c_a and c_b. Write the term which expresses the property which two vectors $\vec{a}$ and $\vec{b}$ need to have.

(2) Find the two constants c_a and c_b which satisfy Eq. ①, when $\vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{a} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$, and $\vec{b} = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$.

(3) Find the real part and the imaginary part of the complex number $e^{i\frac{\pi}{12}}$.

[3] Answer the following questions where $\mathbf{k} = (k_x, k_y, k_z)$, $\mathbf{r} = (x, y, z)$, and $r = |\mathbf{r}|$.

(1) Compute the gradient $\nabla e^{i\mathbf{k}\cdot\mathbf{r}}$.

(2) Prove that $\frac{\partial r}{\partial x} = \frac{x}{r}$, where $r \neq 0$.

(3) Compute the divergence $\mathrm{div}\frac{\mathbf{r}}{r^3}$, where $r \neq 0$.

(4) Evaluate

$$\int \nabla \cdot \left(\frac{\mathbf{r}}{r^3}\right) \mathrm{d}V,$$

where V_a is the spherical area whose center is at the origin and whose radius is a.