¢ E:¶ I –Õ«Äç

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2022 å 7 D 27 Ô

‡QUV

Š{x , ¿aG¶ ØCJ¶ætÖ¶`hÚ™w 4DÍfT' ,6DÍf (H1
»"Ü) t‰è^•"®¢ E:¶ I¯t b"è[w°æ›±Ú"©ßïÓw
-J{q`o–;b"\q›è$q`h‹wpb .ŠR, –;b"-J{x ,
q-Ëœ‰² .³y > *É (ØCJ¶æ-») |g ô|O ˜Ì
²ž¶³ ,•ó,Åy¢ E:¶ , é ¢ 2017£¦ 1§
pb.`T`sU' , ÍGw-J{x , æ»T'•‡"hŠ , ôG€ Ö¼›ß
€` , \w-J{pxÕ«ÄçT'•ŠoM‡b . ^'t , \w-J{p{˜•
"º0x , ¦ïÃÚïÅè[wH 1sT'H 4sütìp`‡b . \w-J{w
G‡TsîÊˆx , Y¦ 3§›€°`oM‡b . \wÔ› "o , ‡cxò
`Í[‡b .
^o, ôs¶Í:¶tSMo , qOwa pxÕ«Äç›®:¶ B¯p{M‡
b.® ØÍwÕ«Äç¯T'•‡" , ®í wÕ«Äç¯ , ^'t , fw¤p®í
w$ ¯›.˜`‡b . fw°Mp , ®Žu¯• , í tSZ"Ú¢• ØwM
Üx{˜sMsr , º0Uv'•oM‡b . \w±Ú"©ßïÓpx , Õ«Ä
çw®Žu¯• , Ú¢• ØwM Ü‹{M‡b . \•'w°æx , ®:¶ B¯w
-J{t'lox , ®C2¯p{˜•oM‡bU , ¿aG¶wx ¶—¼g (²
8¼g) pxc"ŽqsloM‡b . Ú¢• ØwM Ü , ^'txæ»x , aˆ
wôs¶Íw:¶w] pÓèt{˜•oM‡`h . `hUlo , ¶6b"OQ
p, ®å¸¯qMOîxsM‹wqßQoM‡b . bs˜j , \w-J{p{˜•
"º0x , ôs¶Í®:¶ I¯, ®:¶ II¯›¶œpSZy , , Š , grpV"'O
tG\`oM‡b . fw , G¶pwÕ«Äçw¯GO›;M"\qt'" , G
¶tÖ¶`oT'‹µÜ"¶t¶6pV"'OtG\›æloM‡b .
¤º0px , ®«J¯›~œiœt "Ö• , ¶pV"'OtG\`oM‡
b. ^'t Èb"ðJ›®ð¯w Üp)QoM‡b . \•'x , „qœrU¦
æ´ÆçsðJqsloM‡b . ‡hx , °æG¶Ö¼ðJ›~J`o^R`o
M‡b. \•'wð›rX\qt'lo , gr› Š"\qU84^•‡b .
›srt‹ L`oM"wp , ×üU^R`hrtqz±b"\q‹Dópb .
\w-J{t'lo , ØCJ¶æpw:¶t Èb"-Jw®Ú¿½ï¬¯w
OAQ›¬Ýb"qq‹t , •R, ØCJ¶æp¶|hMqMO™IU²Íb"
\q›&loM‡b .

7 ™ t , \ w - J { › ^ R b ” t K h “ , a ™ $ œ t x , Š w Í Y | t - ‰ w ¬ Ý › æ s l o Ö V ‡ ` h . ~ Š o , G s ” ] — › À “ , ° X ò ` Í [‡ b .

2022 å 7 D 27 Ô ¶

è Í

1

Õ « Ä ç

1.1 Ø Õ « Ä ç

1.1.1 Ø Õ « Ä ç w , Å

ô Í : ¶ p ¶ 6 ` h ' O t , Ø Õ « Ä ç $\vec{a}$ x ,

$$\vec{a} = \begin{pmatrix} a_1 & a_2 \end{pmatrix}$$

w ' O t î : a_1, a_2 › ; M o R ü ¯ Ô p V " . \ • x , æ Õ « Ä ç (low vector) q ' • . G ¶ t S M o , Õ « Ä ç x , È p

$$\vec{a} = \boldsymbol{a}$$

w ' O t ¯ G b " . Š ± Ú " © ß ï Ó p ‹ , › t … ' s M v " , È p ¯ G b " . G ¶ p x , » Õ « Ä ç (column vector) q M l o , R ü x , < G w ' O t N t , o ¯ G b " .

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

2 m w Ø Õ « Ä ç

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

q î : k t 0 ` o , è q î : ›

$$\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}, \ k\boldsymbol{a} = \begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}$$

q Š " . ‡ h , b , o w R ü U 0 p K " Õ « Ä ç › µ Õ « Ä ç (zero vector) q ' • . ^ ' t , Ø Õ « Ä ç ¶ . w s b B ù ›

$$\mathbb{R}^2=\left\{\begin{pmatrix}x\\y\end{pmatrix},\ \middle|\ x,\ y\in\mathbb{R}\right\}$$

q S X . Ø Õ « Ä ç $\vec{a}=\boldsymbol{a}=\begin{pmatrix}a_1\\a_2\end{pmatrix}$ w G V ^ (Õ ^) $|\boldsymbol{a}|$ x

$$|\vec{a}|=|\boldsymbol{a}|=\left|\begin{pmatrix}a_1\\a_2\end{pmatrix}\right|=\sqrt{a_1^2+a_2^2}$$

p [^ • " . ô Í : ¶ p x Õ « Ä ç w G V ^ x $|\boldsymbol{a}|$ w ' O t ˆ 0 ‹ G ø › ; M o ¯ b \ q t s l o M " U , \ • T ' x î : w ˆ 0 ‹ q à b " h Š t , Ž < w ' O s G ø › ; M " .

$$\|\boldsymbol{a}\|=\sqrt{a_1^2+a_2^2}$$

\ • › G ¶ p x , Õ « Ä ç w Ê ç Ü (norm) q M O.

\ \ p , G V ^ U 1 p K " Õ « Ä ç › o • Õ « Ä ç (unit vector) q M M, $\boldsymbol{a}$ q ‰ a ² V w o • Õ « Ä ç $\boldsymbol{e}$ x , Ž < w ' O t [^ • " .

$$\boldsymbol{e}=\frac{\boldsymbol{a}}{\|\boldsymbol{a}\|}=\frac{1}{\sqrt{a_1^2+a_2^2}}\begin{pmatrix}a_1\\a_2\end{pmatrix}$$

1.1.2 Ø Õ « Ä ç w º u

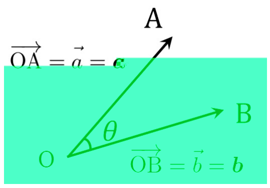


Fig. 1.1

ô Í : ¶ p x , 2 m w Õ « Ä ç $\overrightarrow{\mathrm{OA}}=\vec{a}, \overrightarrow{\mathrm{OB}}=\vec{b}$, w º u (inner product) x , s b ¯ › $\theta, (0\leq\theta\leq\pi)$ « [1] q ` o ,

$$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$$

« [1] G ¶ p x , Æ s Ü ≧, ≦ › ≥, ≤ q M O ' O t , ° Š w s ø p ¯ G b " q`o¯b .`T`sU' ,\•T'xè×wuqw ‰›†Z"hŠt ,G¶ :¶px ,$\overrightarrow{\mathrm{OA}} = \boldsymbol{a}, \overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ q¯G`o ,Ž<w'OsGø›;M" .

$$(\boldsymbol{a}, \boldsymbol{b}) = \|\boldsymbol{a}\| \|\boldsymbol{b}\| \cos\theta$$

2 mw ØÕ«Äç

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

wºux ,

$$(\boldsymbol{a}, \boldsymbol{b}) = a_1 b_1 + a_2 b_2$$

pK" . ‡h , Õ«Äçx æ»(matrix) q_"\q‹DóswP ,fwqVtx ,

$$(\boldsymbol{a}, \boldsymbol{b}) = {}^t\boldsymbol{a}\boldsymbol{b} = \begin{pmatrix} a_1 & a_2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$$

q‹ßQ'•" . \\p ,${}^t\boldsymbol{a}$ x $\boldsymbol{a}$ w8" (transpose) q'y•"‹wp ,

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \ {}^t\boldsymbol{a} = \begin{pmatrix} a_1 & a_2 \end{pmatrix}$$

q`o , [^•" .

ºutmMox , Íw,ŠQíUR"qm\q›ôÍp¶6`h .

ËJ 1.1 ¢ºuwQí £ Ú™w ØÕ«Äç $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ qî: k t0`o ,Ž< UR"qm .

(1) $(\boldsymbol{a}, \boldsymbol{a}) = \|\boldsymbol{a}\|^2$.

(2) $(\boldsymbol{a}+\boldsymbol{b}, \boldsymbol{c}) = (\boldsymbol{a}, \boldsymbol{c}) + (\boldsymbol{b}, \boldsymbol{c}), (\boldsymbol{a}, \boldsymbol{b}+\boldsymbol{c}) = (\boldsymbol{a}, \boldsymbol{b}) + (\boldsymbol{a}, \boldsymbol{c})$.

(3) $(\boldsymbol{a}, \boldsymbol{b}) = (\boldsymbol{b}, \boldsymbol{a})$.

(4) $(k\boldsymbol{a}, \boldsymbol{b}) = (\boldsymbol{a}, k\boldsymbol{b}) = k(\boldsymbol{a}, \boldsymbol{b})$.

(5) $(\boldsymbol{a}, \boldsymbol{a}) \geq 0$ pK"| $(\boldsymbol{a}, \boldsymbol{a}) = 0 \Leftrightarrow \boldsymbol{a} = 0$.

«J 1.1 Ž<wºut b"ALUR"qm\q›Ôd .

$$\|\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}\|^2 = \|\boldsymbol{a}\|^2 + \|\boldsymbol{b}\|^2 + \|\boldsymbol{c}\|^2 + 2(\boldsymbol{a}, \boldsymbol{b}) + 2(\boldsymbol{b}, \boldsymbol{c}) + 2(\boldsymbol{c}, \boldsymbol{a})$$

¬rt- [tHlo-‰ (2‰) ›æO.

$$\|\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}\|^2 = (\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}, \boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$$

q ™ x r b ” .

« J 1.2 : (x_0, y_0) › è “ , Õ « Ä ç $\boldsymbol{a} = \begin{pmatrix} 1 \\ m \end{pmatrix}$ t æ s Ú ¢ w M Ü x $y = m(x - x_0) + y_0 = mx - mx_0 + y_0$ p K ” . î M , \ w ‘ O s Ú ¢ Í w : (x, y) x Í å Ý ” » (parameter) t › ; M o

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t \begin{pmatrix} 1 \\ m \end{pmatrix}$$

q ¯ d ” . \ • ‘ “ , $x = x_0 + t, y = y_0 + mt$ p K ” T ’ , t › « ˆ b • y Ú ¢ w M Ü U ˜ ’ • ” .

« ™1.1 Í å Ý ” » t x , p ! : q ‹ z y • ” .

[1.1 ¢ Ø t S Z ” Ú ¢ w Í å Ý ” » ¯ Ô £ Ø t S Z ” Ú ¢ Í w : › ¯ b Õ « Ä ç $\boldsymbol{x}$ x , Ú ¢ Í w K ” 1 : › ¯ b Õ « Ä ç $\boldsymbol{x}_0$ q M ² › ¯ b M ² Õ « Ä ç $\boldsymbol{v}(= \boldsymbol{0})$ q Í å Ý ” » t › ; M o

$$\boldsymbol{x} = \boldsymbol{x}_0 + t\boldsymbol{v}$$

q ¯ d ” } \ w ‘ O s ¯ Ô › Ú ¢ w Í å Ý ” » ¯ Ô (Ú ¢ w p ! : ¯ Ô) q M O } K ” M x , Ú ¢ w Õ « Ä ç M Ü q M O }

Ú ¢ w Í å Ý ” » ¯ Ô $\boldsymbol{x} = \boldsymbol{x}_0 + t\boldsymbol{v}$ t

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix},\ \boldsymbol{x}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix},\ \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

› E Ö b ” q

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} x_0 + tv_1 \\ y_0 + tv_2 \end{pmatrix}$$

q s ” . \ • ‘ “ t › « ˆ b • y

$$\begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} = t \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Leftrightarrow v_2(x - x_0) = v_1(y - y_0)$$

q s ” w p , $a = v_2, b = -v_1, c = -v_2x_0 + v_1y_0$ q S Z y

$$ax + by + c = 0$$

q ¯ d ” . m ‡ “ Ø º w Ú ¢ w M Ü x x q y w ° Í M Ü q s ” .

« ™1.2 Í … p † Ì b ” 3 Í i í p x , à b ” h Š t ,

$$ax + by + c = 0,\ z = 0$$

q ¯ G b ” . b s ˜ j , 3 Í i í p x , $ax + by + c = 0$ i Z p x Ø w M Ü › ¯ b \ q t « ™ U ž A p K ” .

ð 1.1 2 : A(2,1), B(1,5) › è ” Ú ¢ w Í å Ý ” » ¯ Ô (Õ « Ä ç M Ü) › { Š ‘ .

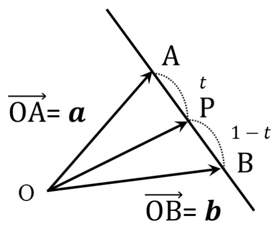


Fig. 1.2

Ú ¢ w Í å Ý ” » ¯ Ô t S Z ” w r q ` o , Fig. 1.2 w ‘ O t , ¢ ü AB › $t : 1-t,\ (0 \le t \le 1)$ t º ü ` o M ” q ß Q • y ,

$$\overrightarrow{OP} = (1-t)\overrightarrow{OA} + t\overrightarrow{OB} = \overrightarrow{OA} + t\overrightarrow{AB}$$

$$\Leftrightarrow \boldsymbol{p} = \boldsymbol{a} + t(\boldsymbol{b} - \boldsymbol{a})$$

p K “ , Í å Ý ” » ¯ Ô q ° • b ” .

« ™1.3 $t < 0, 1 < t$ ‡ p ¦ Á b • y , ¢ ü AB T ’ Ú ¢ AB › ¯ b \ q t s ” . \ \ p , t › > t Ì q ˆ s d y , $\overrightarrow{OA}$ x , $t = 0$ w q V , : A, $t = \frac{1}{2}$ w q V , ¢ ü AB w ¤ : , $t = 1$ w q V , : B › ¦ b q r b ” \ q U Z R ” .

« ™1.4 Ÿ s ” 2 : A(x_1, y_1), B(x_2, y_2) (h i ` , $x_1 \ne x_2$ T m $y_1 \ne y_2$) › è ” Ú ¢ U , ô Í : ¶ p x

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ or } y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \text{ or } \frac{x - x_2}{x_2 - x_1} = \frac{y - y_2}{y_2 - y_1}$$

q G \ ^ • o M ” . ` T ` , \ w ¯ G t x , $x_1 \ne x_2, y_1 \ne y_2$ › ž A q b ” . f \ p , M ² Õ « Ä ç $\boldsymbol{v} = \overrightarrow{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$ › ‹ Ö b • y ,

$$\boldsymbol{p} = \overrightarrow{OA} + t\overrightarrow{AB} = \overrightarrow{OA} + t\boldsymbol{v}$$

$$\Leftrightarrow \ \boldsymbol{p} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + t \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} \ \text{or} \ \boldsymbol{p} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + t \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

q ¯ G b ” M U Š í $ p K ” .

1.1.4 Y ù è Õ « Ä ç

j : O q b ” 2 ª Ø Í t Ÿ s ” 2 : A, B › ß Q ” . ‡ h , $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}, \overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ q b ” . h i ` , $\angle\mathrm{AOB} = \theta, 0 < \theta < \frac{\pi}{2}$ q b ” . : A T ’ : O › • : q b ” R Ú ¢ OB t < ` h (¢ q R Ú ¢ OB q w ¦ : › H q b ” . \ w q V , $\overrightarrow{\mathrm{OH}} = \boldsymbol{h}$ › Y ù è Õ « Ä ç (orthographic projection vector) q M O. Y ù è Õ « Ä ç x , ¬ å Ü ~ ³ á Û ¿ Ä w Y F Ú ¦ = O (Gram-Schmidt orthonormalization) p b ; ^ • ” .

« J 1.3 Y ù è Õ « Ä ç U Ž < p ¯ ^ • ” \ q › Ô d .

$$\overrightarrow{\mathrm{OH}} = \boldsymbol{h} = \frac{(\boldsymbol{a}, \boldsymbol{b})}{\|\boldsymbol{b}\|^2}\boldsymbol{b}$$

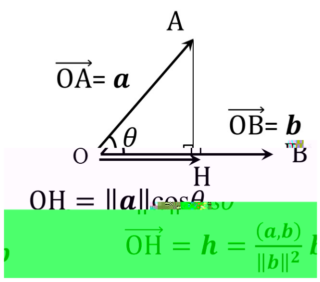


Fig. 1.3

¬ r t - ‡ c , $\mathrm{OH} = \|\boldsymbol{a}\|\cos\theta$ p K ” . ^ ’ t , $\boldsymbol{b}$ w o • Õ « Ä ç t OH b • y ‘ M w p ,

$$\overrightarrow{\mathrm{OH}} = \boldsymbol{h} = \|\boldsymbol{a}\|\cos\theta \cdot \frac{\boldsymbol{b}}{\|\boldsymbol{b}\|} = \frac{\|\boldsymbol{a}\|\|\boldsymbol{b}\|\cos\theta}{\|\boldsymbol{b}\|^2}\boldsymbol{b} = \frac{(\boldsymbol{a}, \boldsymbol{b})}{\|\boldsymbol{b}\|^2}\boldsymbol{b} \qquad \square$$

[1.2 ¢ Ø Í w Ú ¢ w Õ « Ä ç M Ü £ Ø t S Z ” Ú ¢ Í w : › ¯ b Õ « Ä ç $\boldsymbol{x}$ x | Ú ¢ Í w K ” 1 : › ¯ b Õ « Ä ç $\boldsymbol{x}_0$ q Ú ¢ q (Ú s Õ « Ä ç $\boldsymbol{n}(\neq \boldsymbol{0})$ › ; M o

$$(\boldsymbol{x} - \boldsymbol{x}_0, \boldsymbol{n}) = 0$$

q ¯ d ” } $\boldsymbol{n}$ › \ w Ú ¢ w O ¢ Õ « Ä ç q M O }

« ™1.5 Ú ¢ w Í å Ý ” » ¯ Ô $\boldsymbol{x} = \boldsymbol{x}_0 + t\boldsymbol{v}$ › Õ « Ä ç M Ü q z œ i U , $(\boldsymbol{x} - \boldsymbol{x}_0, \boldsymbol{n}) = 0$ ‹ Ú ¢ w Õ « Ä ç M Ü q ‘ • .

Õ « Ä ç M Ü $(\boldsymbol{x} - \boldsymbol{x}_0, \boldsymbol{n}) = 0$ t $\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \boldsymbol{x}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}, \boldsymbol{n} = \begin{pmatrix} a \\ b \end{pmatrix}$ › E Ö b ” q

$$0 = (x - x_0)a + (y - y_0)b = ax + by - ax_0 - by_0$$

q s ” w p , $c = -ax_0 - by_0$ q S Z y

$$ax + by + c = 0$$

q ¯ d ” . ‘ l o , Ø Í w Ú ¢ ‹ x q y w ° Í M Ü p ¯ ` h q V , x, y w : a, b U Ú ¢ w O ¢ Õ « Ä ç w R ü q s l o M ” \ q U ˜ T ” . b s ˜ j , $\boldsymbol{n} = \begin{pmatrix} a \\ b \end{pmatrix}$ p K ” .

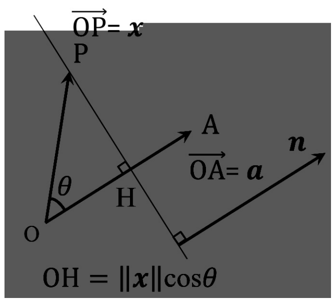


Fig. 1.4

o t , Õ « Ä ç $\boldsymbol{a} \neq 0$ ›) Q o , $\boldsymbol{a}$ q w º u U º p K ” : P w J { › ß Q ” . b s ˜ j , $(\boldsymbol{x}, \boldsymbol{a}) = C$, ($C$ x :) p K ” q V , $\boldsymbol{x}$ w J { › { Š ” . Fig. 1.4 t S M o , A › $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$ t q “ , $\overrightarrow{\mathrm{OP}} = \boldsymbol{x}$ q b ” .

\ w q V , $\boldsymbol{x}_0 = \overrightarrow{\mathrm{OH}}$ q b ” q $\left\|\overrightarrow{\mathrm{OP}}\right\| \cos\theta = \mathrm{OH}$ t £ è ` o ,

$$(\boldsymbol{x}_0, \boldsymbol{a}) = \left\|\overrightarrow{\mathrm{OH}}\right\| \cdot \left\|\overrightarrow{\mathrm{OA}}\right\| = \left\|\overrightarrow{\mathrm{OP}}\right\| \cdot \left\|\overrightarrow{\mathrm{OA}}\right\| \cos\theta$$
$$= (\boldsymbol{x}, \boldsymbol{a}) = C$$

` h U l o ,

$$(\boldsymbol{x}-\boldsymbol{x}_0,\boldsymbol{a})=0$$

q s “ , $\boldsymbol{a}$ › O ¢ Õ « Ä ç t Ë m Ú ¢ › ¯ b .

ð 1.2 j : O q b ” 2 ª Ø › ß Q ” . Ú ¢ $y=-x+2$ Í w ˆ : P, S ‘ | : A(2,2) t 0 ` o , º u $\left(\overrightarrow{\mathrm{OP}},\overrightarrow{\mathrm{OA}}\right)$ U ° p K ” \ q › Ô d .

« J 1.4 $\triangle$OAB t S M o , OA= 5, OB= 8, AB= 7 q b ” . $\triangle$OAB w (ú › H q b ” . $\overrightarrow{\mathrm{OH}}=\boldsymbol{h}$ › $\overrightarrow{\mathrm{OA}}=\boldsymbol{a}$, $\overrightarrow{\mathrm{OB}}=\boldsymbol{b}$ › ; M o ¯ d .

¬ r t - º u $(\boldsymbol{a},\boldsymbol{b})$ › { Š ” .

$$\begin{aligned}
&\mathrm{AB}^2=\|\boldsymbol{b}-\boldsymbol{a}\|^2=\|\boldsymbol{a}\|^2-2(\boldsymbol{a},\boldsymbol{b})+\|\boldsymbol{b}\|^2\\
\Leftrightarrow\ &49=25-2(\boldsymbol{a},\boldsymbol{b})+64\\
\Rightarrow\ &(\boldsymbol{a},\boldsymbol{b})=20
\end{aligned}$$

‡ h , $\boldsymbol{h}$ x î : s, t › b ; ` o ,

$$\boldsymbol{h}=s\boldsymbol{a}+t\boldsymbol{b}$$

q G \ p V ” . \ \ p , Ú ¢ AH Í w Ú ™ w : P t 0 ` o , º u $\left(\boldsymbol{b},\overrightarrow{\mathrm{OP}}\right)$ x ° p K ” . b s ˜ j ,

$$\left(\boldsymbol{b},\overrightarrow{\mathrm{OP}}\right)=(\boldsymbol{b},\boldsymbol{h})=(\boldsymbol{a},\boldsymbol{b})$$

U R “ q m . ‰ 7 t , Ú ¢ BH Í w Ú ™ w : Q t 0 ` o , º u $\left(\boldsymbol{a},\overrightarrow{\mathrm{OQ}}\right)$ ‹ ° p K ” . Ž Í ‘ “ ,

$$\left(\boldsymbol{a},\overrightarrow{\mathrm{OQ}}\right)=(\boldsymbol{a},\boldsymbol{h})=(\boldsymbol{a},\boldsymbol{b})$$

7 4 $ t ,

$$(\boldsymbol{a},\boldsymbol{b})=(\boldsymbol{a},\boldsymbol{h})=(\boldsymbol{b},\boldsymbol{h})$$

U R “ q m . Ž Í ‘ “ ,

$$\begin{cases}
(\boldsymbol{a},\boldsymbol{b})=(\boldsymbol{a},\boldsymbol{h}) & \Rightarrow\ s\|\boldsymbol{a}\|^2+t(\boldsymbol{a},\boldsymbol{b})=(\boldsymbol{a},\boldsymbol{b}) & \Rightarrow\ 25s+20t=20\\
(\boldsymbol{a},\boldsymbol{b})=(\boldsymbol{b},\boldsymbol{h}) & \Rightarrow\ s(\boldsymbol{a},\boldsymbol{b})+t\|\boldsymbol{b}\|^2=(\boldsymbol{a},\boldsymbol{b}) & \Rightarrow\ 20s+64t=20
\end{cases}$$

$$\Rightarrow (s,t)=\left(\frac{11}{15},\frac{1}{12}\right)$$

` h U l o ,

$$h = \frac{11}{15}a + \frac{1}{12}b \qquad \square$$

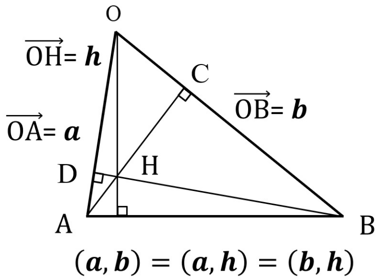


Fig. 1.5

r q ` o , Y ù è Õ « Ä ç › b ; b ” . Fig. 1.5 w ‘ O t 2 : C, D › q ” . : C x : A w Y ù è , : D x : B w Y ù è s w p ,

$$\overrightarrow{\mathrm{OC}} = \frac{(a,b)}{\|b\|^2}b = \frac{5}{16}b,\ \overrightarrow{\mathrm{OD}} = \frac{(a,b)}{\|a\|^2}a = \frac{4}{5}a$$

° M , AH:HC=$\alpha : 1-\alpha$, BH:HD=$\beta : 1-\beta$ t º ü b ” q ß Q • y ,

$$h = \alpha\overrightarrow{\mathrm{OC}} + (1-\alpha)a = (1-\alpha)a + \frac{5\alpha}{16}$$

OD(1 ∈ β a △ ′ / 4 aad

« ™1.6 Ý É å ¢ µ w g ' " ,

$$\frac{\mathrm{AD}}{\mathrm{DO}}\cdot\frac{\mathrm{OB}}{\mathrm{BC}}\cdot\frac{\mathrm{CH}}{\mathrm{HA}}=\frac{1}{4}\cdot\frac{16}{11}\cdot\frac{1-\alpha}{\alpha}=1 \Rightarrow \alpha=\frac{4}{15}$$

q { Š " \ q ‹ D ó p K " .

ð 1.3 $\triangle$OAB t S M o , OA= 4, OB= 3, $\angle$OAB= $\frac{\pi}{3}$ q b " . $\triangle$OAB w (ú › H q b " . $\overrightarrow{\mathrm{OH}}=\boldsymbol{h}$ › $\overrightarrow{\mathrm{OA}}=\boldsymbol{a}$, $\overrightarrow{\mathrm{OB}}=\boldsymbol{b}$ › ; M o ¯ d .

1.1.5 Ø t S Z " w M Ü

Ø Í w x , ¤ ú q R p ‡ " .

« J 1.5 ¤ ú (a,b), R $r(>0)$ w M Ü x $(x-a)^2+(y-b)^2=r^2$ p K " . î M , \ w ' O s * Í w : (x,y) x Í å Ý " » $\theta, (0\le\theta<2\pi)$ › ; M o

$$\boldsymbol{x}=\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}+\begin{pmatrix}r\cos\theta\\r\sin\theta\end{pmatrix}$$

q ¯ d " . \ • ' " , $x=a+r\cos\theta$, $y=b+r\sin\theta$ p K " T ' , θ › « ˆ b • y w M Ü U ˜ ' • " . \ w ¯ Ô x , w Í å Ý " » ¯ Ô q z y • " .

[1.3 ¢ w Õ « Ä ç M Ü £ Ø t S Z " * Í w : › ¯ b Õ « Ä ç $\boldsymbol{x}$ x , ¤ ú w • " Õ « Ä ç $\boldsymbol{a}$ q R r › ; M o

$\|\boldsymbol{x}-\boldsymbol{a}\|=r$ w :°<@>päÐÁ@.•À

2

w ' O t î : a_1, a_2, a_3 › ; M o R ü ¯ Ô p V ” . 2 m w í Õ « Ä ç

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

q î : k t 0 ` o , è q î : ›

$$\boldsymbol{a}+\boldsymbol{b} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{pmatrix}, \ k\boldsymbol{a} = \begin{pmatrix} ka_1 \\ ka_2 \\ ka_3 \end{pmatrix}$$

q Š ” . \ • ' x , z R ü U ÿ Q h Ž Ž , Ø Õ « Ä ç w , Å 1.1.1 p Š h ‰ q ¶ X ‰ 7 p K ” .

Ø Õ « Ä ç ‰ 7 t , í Õ « Ä ç $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ w Ê ç Ü (G V ^) $\|\boldsymbol{a}\|$ x

$$\|\boldsymbol{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

p [^ • ” . ‡ h , $\boldsymbol{a}$ q ‰ a ² V w o • Õ « Ä ç $\boldsymbol{e}$ x , Ž < w ' O t [^ • ” .

$$\boldsymbol{e} = \frac{\boldsymbol{a}}{\|\boldsymbol{a}\|} = \frac{1}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

1.2.2 í Õ « Ä ç w º u

Ø Õ « Ä ç ‰ 7 t , 2 m w Õ « Ä ç $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}, \overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ w º u x , s b ¯ › θ, $(0 \le \theta \le \pi)$ q ` o ,

$$(\boldsymbol{a}, \boldsymbol{b}) = \|\boldsymbol{a}\| \|\boldsymbol{b}\| \cos\theta$$

w ' O t ¯ b . 2 m w í Õ « Ä ç

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

w º u x ,

$$(\boldsymbol{a}, \boldsymbol{b}) = a_1 b_1 + a_2 b_2 + a_3 b_3$$

p K ” .

º u t m M o x , Ø Õ « Ä ç ‰ 7 , Ë J 1.1 U ‰ 7 t R “ q m .

« J 1.6 í w 2 m w Õ « Ä ç $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ w s b ¯ $\theta, (0 < \theta < \pi)$ › { Š ‘ .

¬ r t - ‘ X Œ ’ • o M ” ‘ O t , Ž < w ¬ Ü U R “ q m .

$$\cos\theta = \frac{(\boldsymbol{a}, \boldsymbol{b})}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|}$$

` h U l o ,

$$\cos\theta = \frac{(\boldsymbol{a}, \boldsymbol{b})}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|} = \frac{7}{\sqrt{14} \cdot \sqrt{14}} = \frac{1}{2}$$

` h U l o , $\theta = \frac{\pi}{3}$. □

ð 1.5 î : t t 0 ` o , 3 : A(2,4,0), B(0,2,0), P$(0, 4 - t^2, 2t)$ › Š ” . \ w q V , ∠BAP › { Š ‘ . (1987 G U G ¶ (~ J))

1.2.3 Ž u

2 m w Õ « Ä ç $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}, \overrightarrow{\mathrm{OB}} = \boldsymbol{b}$, w Ž u (cross product) › [b ” .

Fig. 1.6

[1.4 ¢ Ž u £ 2 m w Õ « Ä ç $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}, \overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ w Ž u ›

$$\overrightarrow{\mathrm{OA}} \times \overrightarrow{\mathrm{OB}} = \boldsymbol{a} \times \boldsymbol{b}$$

q G \ b ” . \ w q V ,

(0) $\boldsymbol{a}, \boldsymbol{b}$ w — s X q ‹ ° M U $\boldsymbol{0}$ w q V , $\boldsymbol{a} \times \boldsymbol{b} = 0$ q Š ” .

(1) $\boldsymbol{a}, \boldsymbol{b}$ U q ‹ t $\boldsymbol{0}$ p s M q V , $\boldsymbol{a} \times \boldsymbol{b}$ x , $\boldsymbol{a}, \boldsymbol{b}$ w † M t (Ú s Õ « Ä ç p K ” . f w ² V x , $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{a} \times \boldsymbol{b}$ w q p È % q s ” . b s ˜ j , : A T ’ : B t ² T O ‘ O t s ` h q V , v a U ‰ M ² p K ” .

(2) $\boldsymbol{a} \times \boldsymbol{b}$ w G V ^ $\|\boldsymbol{a} \times \boldsymbol{b}\|$ x , $\boldsymbol{a}, \boldsymbol{b}$ w s b ¯ › $\theta, (0 \leq \theta \leq \pi)$ q ` o ,

$$\|\boldsymbol{a} \times \boldsymbol{b}\| = \|\boldsymbol{a}\| \|\boldsymbol{b}\| \sin\theta \tag{1.1}$$

(3) 2 m w í Õ « Ä ç

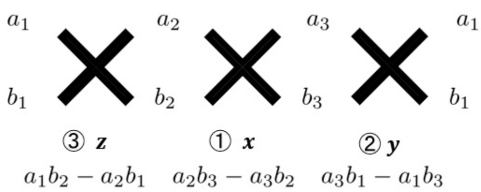
$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

w Ž u $\boldsymbol{a} \times \boldsymbol{b}$ w R ü ¯ Ô x ,

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \tag{1.2}$$

p K ” .

« ™1.7 Ž u w - ‰ x , Ž < w ‘ O t ® Q o S X q (b p K ” .



Ë J 1.4 ¢ Ž u w Q í £ Ú ™ w í Õ « Ä ç $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ q î : k t 0 ` o , Ž < U R “ q m .

(1) $\boldsymbol{a} \times \boldsymbol{a} = 0$.

(2) $(\boldsymbol{a}+\boldsymbol{b})\times\boldsymbol{c}=\boldsymbol{a}\times\boldsymbol{c}+\boldsymbol{b}\times\boldsymbol{c},\ \boldsymbol{a}\times(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{a}\times\boldsymbol{c}.$

(3) $\boldsymbol{a}\times\boldsymbol{b}=-\boldsymbol{b}\times\boldsymbol{a}.$

(4) $(k\boldsymbol{a})\times\boldsymbol{b}=\boldsymbol{a}\times(k\boldsymbol{b})=k(\boldsymbol{a}\times\boldsymbol{b}).$

« ™1.8 (3) x A « ™ p K ” . b s ˜ j , Ž u w ¦ õ O x R “ q h c , q j › ! Ë b ” q, ² V U o t s ” .

Í t , é . $ s « J › è ` o , Q í › ¬ Ý b ” .

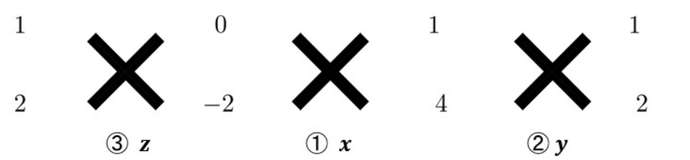
> « J 1.7 3 : A(1,1,4), B(2,1,5), C(3,−1,8) p K ” q V , △ABC w Ø u S › { Š ‘ .

¬ r t - Ž u w Q í (1.1) › b ; b ” . b s ˜ j , AB, AC › 2 % q b ” æ › % w Ø u w $\frac{1}{2}$ U S p K ” \ q › b ; b ” . ‡ c , 2 % › ¯ b Õ « Ä ç x ,

$$\overrightarrow{AB}=\boldsymbol{b}=\begin{pmatrix}1\\0\\1\end{pmatrix},\ \overrightarrow{AC}=\boldsymbol{c}=\begin{pmatrix}2\\-2\\4\end{pmatrix}$$

` h U l o , Ø u S x ,

$$S=\frac{1}{2}\|\boldsymbol{b}\|\|\boldsymbol{c}\|\sin\theta=\frac{1}{2}\|\boldsymbol{b}\times\boldsymbol{c}\|=\frac{1}{2}\left|2\begin{pmatrix}1\\-1\\-1\end{pmatrix}\right|=\sqrt{3}\qquad\square$$



$1\cdot(-2)-0\cdot2=-2\quad 0\cdot4-1\cdot(-2)=2\quad 1\cdot2-1\cdot4=-2$

« ™1.9 ô Í : ¶ p x , í º w △ABC w Ø u S x , Ž < w ¬ Ü › b ; b ” .

$$S=\frac{1}{2}\sqrt{\left|\overrightarrow{AB}\right|^2\left|\overrightarrow{AC}\right|^2-\left(\overrightarrow{AB}\cdot\overrightarrow{AC}\right)^2}\tag{1.3}$$

\ • › b ; b • y ,

$$S=\frac{1}{2}\sqrt{\sqrt{2}^2\cdot(2\sqrt{6})^2-6^2}=\sqrt{3}$$

q { Š ” \ q U Z R ” . ` T ` s U ’ , Ž < w ð J

® î : x t 0 ` o , $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} x \\ x+1 \\ x-1 \end{pmatrix}, \overrightarrow{\mathrm{OB}} = \begin{pmatrix} x-1 \\ x \\ x+1 \end{pmatrix}$ p K ” q V , $\triangle\mathrm{OAB}$ w Ø u S w 7 – ‹ › { Š ‘ . ¯

p x , Ž u w b ; U y W $ t b t s ” . î M ,

$$S = \frac{1}{2}\left\|\overrightarrow{\mathrm{OA}} \times \overrightarrow{\mathrm{OB}}\right\| = \frac{1}{2}\left|\begin{pmatrix} 3x+1 \\ -3x+1 \\ 1 \end{pmatrix}\right| = \frac{1}{2}\sqrt{18x^2+3}$$

‘ “ , $x = 0$ w q V , 7 – ‹ $S \geq \frac{\sqrt{3}}{2}$ › q ” \ q U 0 › t Ô ^ • ” .

ð 1.6 3 : O(0,0,0), P(1,0,a), Q(0,2,b) p K ” q V , $\triangle\mathrm{OPQ}$ w Ø u S › a, b › ; M o ¯ d .

« J 1.8 2 m w í Õ « Ä ç $\boldsymbol{a} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ w † M [illegible] 13 q .48.2491 Tf -20.483-15.093 Td<0f08F6 8.2491

w Õ « Ä ç $\boldsymbol{e}$ › { Š ‘ . 13 q .48.2491 Tf -2- S 1 q ./F15 4.1656(Tf -20.483-15

13 q .48.2491 Tf -20.483-15.093 Td6TJ79F6 8.2491 Tf

¬ r t - Ž u w - ‰ Ü (1.2) › b ; b ” . [illegible] 6 0 Td[(a)]TJ6203 9.1656 Tf 6.965 0 Td[(Æ)]T893F15 9

d6TJ

ð 1.7 2 m w í Õ « Ä ç $\boldsymbol{a}=\begin{pmatrix}-1\\1\\1\end{pmatrix}, \boldsymbol{b}=\begin{pmatrix}2\\1\\0\end{pmatrix}$ w † M t (Ú s o • Õ « Ä ç › { Š ‘ .

1.2.4 µ § å ” ~ O u

Ž < w ‘ O s æ á Ø . w . u V › { Š ‘ O .

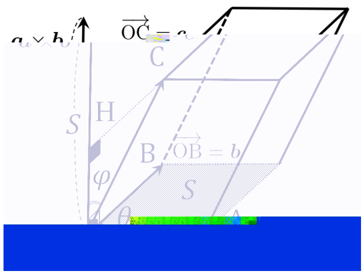


Fig. 1.7

Fig. 1.7 ‘ “ , $\boldsymbol{a}, \boldsymbol{b}$ › 2 % q b ” æ › % w Ø u › S q ` o ,

$$V = S\cdot \mathrm{OH} = \|\boldsymbol{a}\|\|\boldsymbol{b}\|\sin\theta\cdot\|\boldsymbol{c}\||\cos\phi| = \|\boldsymbol{a}\times\boldsymbol{b}\|\cdot\|\boldsymbol{c}\||\cos\phi|$$
$$= \left|\left(\boldsymbol{a}\times\boldsymbol{b},\boldsymbol{c}\right)\right|$$

p) Q ’ • ” . › t , $(\boldsymbol{a}\times\boldsymbol{b},\boldsymbol{c})$ › µ § å ~ O u (scalar triple product) q M O.
™ t { O æ » Ü (determinant) › b ; b • y ,

$$\boldsymbol{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}, \boldsymbol{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}, \boldsymbol{c}=\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix}$$

q ` o ,

$$\left(\boldsymbol{a}\times\boldsymbol{b},\boldsymbol{c}\right)=\begin{vmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{vmatrix}$$
$$= c_1\begin{vmatrix}a_2&a_3\\b_2&b_3\end{vmatrix} - c_2\begin{vmatrix}a_1&a_3\\b_1&b_3\end{vmatrix} + c_3\begin{vmatrix}a_1&a_2\\b_1&b_2\end{vmatrix}$$
$$= a_1b_2c_3+a_2b_3c_1+a_3b_1c_2-(a_3b_2c_1+a_1b_3c_2+a_2b_1c_3)$$

« J 1.9 4 : A(1, −1, 3), B(2, 1, 0), C(1, −1, 5), D(−3, 1, 2) p K ” q V , › Ø . ABCD w . u V › { Š ‘ .

¬ r t - µ § å 3 O u › b ; b ” . b s ˜ j , AB, AC, AD › 3 % q b ” æ á Ø . w . u w $\frac{1}{6}$ U V p K ” \ q › b ; b ” . ‡ c , 3 % › ¯ b Õ « Ä ç x ,

$$\overrightarrow{AB} = \boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix},\ \overrightarrow{AC} = \boldsymbol{c} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix},\ \overrightarrow{AD} = \boldsymbol{d} = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix}$$

` h U l o , . u V x ,

$$V = \frac{1}{6}\left|(\boldsymbol{b}\times\boldsymbol{c}, \boldsymbol{d})\right| = \frac{1}{6}\left|\left(\begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix}\right)\right| = \frac{10}{3}$$ □

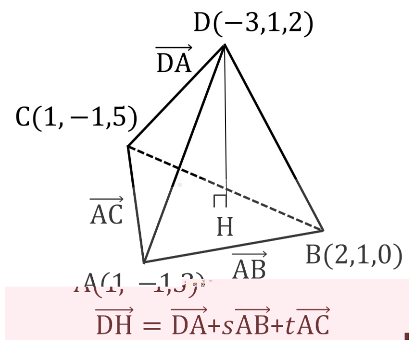


Fig. 1.8

« ™ 1.10 ô Í : ¶ w c “ p r X w p K • y , Ž < w q q s ” . ‡ c , △ ABC w Ø u › (1.3) t ‘ l o - ‰ b ” . $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ s w p ,

$$S = \frac{1}{2}\sqrt{\left|\overrightarrow{AB}\right|^2\left|\overrightarrow{AC}\right|^2 - \left(\overrightarrow{AB}\cdot\overrightarrow{AC}\right)^2} = \sqrt{5}$$

° M , Ö : D T ’ △ ABC • < ` h (¢ w › H(x, y, z) q b • y , 2 m w î : s, t › b ;

` o ,

$$\overrightarrow{DH} = \overrightarrow{DA} + s\overrightarrow{AB} + t\overrightarrow{AC} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 4+s \\ -2+2s \\ 1-3s+2t \end{pmatrix}$$

w ‘ O t ¯ q ^ • ” « 3. \ \ p , $\overrightarrow{DH}$ x , $\overrightarrow{AB}, \overrightarrow{AC}$ w † M t (Ú s w p ,

$$\overrightarrow{DH} \cdot \overrightarrow{AC} = \begin{pmatrix} 4+s \\ -2+2s \\ 1-3s+2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 14s - 6t - 3 = 0$$

$$\overrightarrow{DH} \cdot \overrightarrow{BC} = \begin{pmatrix} 4+s \\ -2+2s \\ 1-3s+2t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = 2(1-3s+2t) = 0$$

\ • ’ › r M o , $s = 0, t = -\frac{1}{2}$ › ˜ ” . ` h U l o , $\overrightarrow{DH} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$. b s ˜ j , $\left\|\overrightarrow{DH}\right\| = 2\sqrt{5}$.

Ž í ‘ “ , { Š ” . u V x ,

$$V = \frac{1}{3}\left\|\overrightarrow{DH}\right\| \times \triangle ABC = \frac{1}{3} \cdot 2\sqrt{5} \cdot \sqrt{5} = \frac{10}{3} \qquad \square$$

\ w ‘ O t , G ! s - ‰ › § M ’ • ” .

ð 1.8 4 : A(2,1,2), B(2,3,6), C(3,6,2) p K ” q V , › Ø . OABC w . u V › { Š ‘ .

1.2.5 Õ « Ä ç ~ O u

3 m w í Õ « Ä ç $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ t 0 ` o ,

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$$

› Õ « Ä ç ~ O u (vector triple product) q M O.

Ë J 1.5 ¢ Õ « Ä ç ~ O u w Q í £ Ú ™ w í Õ « Ä ç $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ t 0 ` o , Ž < U R “ q m .

(1) $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a}, \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}, \boldsymbol{b})\boldsymbol{c}$.

(2) $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{0}$.

¬ Â Ì - (1) w ˆ Ô b . 2 m w í Õ « Ä ç

$$\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \ \boldsymbol{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

« 3 \ • x , ™ { ˜ • ” í º w Ø w Õ « Ä ç M Ü p K ” .

wŽux ,

$$\boldsymbol{b} \times \boldsymbol{c} = \begin{pmatrix} b_2c_3 - b_3c_2 \\ b_3c_1 - b_1c_3 \\ b_1c_2 - b_2c_1 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

pK” . `hUlo ,

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \begin{pmatrix} a_2d_3 - a_3d_2 \\ a_3d_1 - a_1d_3 \\ a_1d_2 - a_2d_1 \end{pmatrix}$$

\\p , ¤Rü›-‰b•y ,

$$\begin{aligned} a_2d_3 - a_3d_2 &= a_2(b_1c_2 - b_2c_1) - a_3(b_3c_1 - b_1c_3) \\ &= (a_2c_2 + a_3c_3)b_1 - (a_2b_2 + a_3b_3)c_1 \\ &= (a_1c_1 + a_2c_2 + a_3c_3)b_1 - (a_1b_1 + a_2b_2 + a_3b_3)c_1 \\ &= (\boldsymbol{a}, \boldsymbol{c})b_1 - (\boldsymbol{a}, \boldsymbol{b})c_1 \\ a_3d_1 - a_1d_3 &= a_3(b_2c_3 - b_3c_2) - a_1(b_1c_2 - b_2c_1) \\ &= (a_1c_1 + a_3c_3)b_2 - (a_1b_1 + a_3b_3)c_2 \\ &= (a_1c_1 + a_2c_2 + a_3c_3)b_2 - (a_1b_1 + a_2b_2 + a_3b_3)c_2 \\ &= (\boldsymbol{a}, \boldsymbol{c})b_2 - (\boldsymbol{a}, \boldsymbol{b})c_2 \\ a_1d_2 - a_2d_1 &= a_1(b_3c_1 - b_1c_3) - a_2(b_2c_3 - b_3c_2) \\ &= (a_1c_1 + a_2c_2)b_3 - (a_1b_1 + a_2b_2)c_3 \\ &= (a_1c_1 + a_2c_2 + a_3c_3)b_3 - (a_1b_1 + a_2b_2 + a_3b_3)c_3 \\ &= (\boldsymbol{a}, \boldsymbol{c})b_3 - (\boldsymbol{a}, \boldsymbol{b})c_3 \end{aligned}$$

‘loÂÌUìƒb” . □

1.2.6 í tSZ”Ú¢wM Ü

ØÍwÚ¢‰7 , Ú¢Íw 1 :q , M²Õ«ÄçU>‡•y 1 mt ‡” .

[1.5 ¢í tSZ”Ú¢wÍåÝ”»¯Ô £ í tSZ”Ú¢Íw: ›¯bÕ«Äç $\boldsymbol{x}$ x, Ú¢ÍwK” 1 :›¯bÕ«Äç $\boldsymbol{x}_0$ qM²›¯b M

² Õ « Ä ç $\boldsymbol{v}(\neq \mathbf{0})$ q Í å Ý " » t › ; M o

$$\boldsymbol{x} = \boldsymbol{x}_0 + t\boldsymbol{v} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

q ¯ d " }

$\boldsymbol{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ q ` o , [' " t › « ˆ b • y

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

q G \ ^ • o M " . ` T ` , \ w ¯ G t x , $a \neq 0, b \neq 0, c \neq 0$ › ž A q b " .

ð 1.9 2 : A(−1,1,0), B(0,−1,1) › è " Ú ¢ w Í å Ý " » ¯ Ô › { Š ' . ^ ' t , Ú ¢ w M Ü › { Š ' .

« J 1.10 : A(2,1,6) T ' , Ú ¢ $\dfrac{x-5}{-2} = \dfrac{y}{3} = z - 1$ • < ` h (¢ w › H q b " . : H › { Š ' .

¬ r t - : H x Ú ¢ Í t K " w p , Í å Ý " » t › ‹ Ö ` ,

$$\frac{x-5}{-2} = \frac{y}{3} = z - 1 = t$$

q S X \ q t ' " ,

$$\overrightarrow{\mathrm{OH}} = \begin{pmatrix} -2t + 5 \\ 3t \\ t + 1 \end{pmatrix}$$

\ w q V , $\overrightarrow{\mathrm{AH}}$ q Ú ¢ w M ² Õ « Ä ç $\boldsymbol{v} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ U (Ú s w p ,

$$\left(\overrightarrow{\mathrm{AH}}, \boldsymbol{v}\right) = \left(\begin{pmatrix} -2t + 3 \\ 3t - 1 \\ t - 5 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \right) = 14t - 14 = 0 \Rightarrow t = 1$$

` h U l o , H(3,3,2). □

r q ` o , $\left\|\overrightarrow{\mathrm{AH}}\right\|^2$ U 7 – q s ” q V , J ™ › ¬ h b .

$$\left\|\overrightarrow{\mathrm{AH}}\right\|^2 = (-2t+3)^2 + (3t-1)^2 + (t-5)^2 = 14(t-1)^2 + 21$$

` h U l o , $t=1$ w q V 7 – q s ” . \ w q V , H(3,3,2) T m 7 – ‹ x $\sqrt{21}$ p K ” . □

ð 1.10 2 : A(2,1,0), B(1,1,1) › è ” Ú ¢ ℓ Í t ˆ : P › q ” . ¢ ü OP w 7 – ‹ › { Š ‘ .

« J 1.11 A(1,0,1), B(0,−1,0) › è ” Ú ¢ ℓ q C(−1,0,3), D(−4,−1,4) › è ” Ú ¢ m U 1 : p ¦ ˜ ” \ q › Ô ` , f w ¦ : › { Š ‘ .

¬ r t - Ú ¢ AB, Ú ¢ CD x , f • g • Í å Ý ” » s, t › b ; ` o ,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overrightarrow{\mathrm{OA}} + s\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1-s \\ -s \\ 1-s \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overrightarrow{\mathrm{OC}} + t\overrightarrow{\mathrm{CD}} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1-3t \\ -t \\ 3+t \end{pmatrix}$$

‹ ` ¦ : U O b ” s ’ ,

$$\begin{pmatrix} 1-s \\ -s \\ 1-s \end{pmatrix} = \begin{pmatrix} -1-3t \\ -t \\ 3+t \end{pmatrix}$$

U ¶ o R “ q m . f \ p , x R ü y R ü t £ è ` ,

$$\begin{cases} 1-s = -1-3t \\ -s = -t \end{cases} \Rightarrow (s,t) = (-1,-1)$$

\ • x , z R ü $1-s = 3+t = 2$ › ¬ h b . ` h U l o , ¦ : U O ` , f w ¦ : x , (2,1,2). □

« J 1.12 Ú ¢ $\ell : x+y=-1,\ z=0$ Í t ˆ : P, Ú ¢ $m : x=y=-\dfrac{z-2}{2}$ Í t ˆ : Q U K ” . ¢ ü PQ w 7 – ‹ › { Š ‘ .

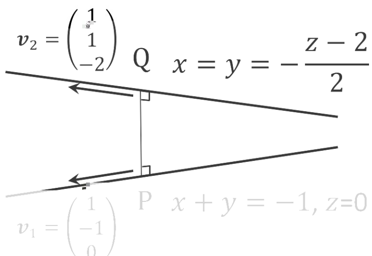


Fig. 1.9

¬ r t - ˆ : P, ˆ : Q x , f • g • Í å Ý ” » s, t › b ; ` o ,

$$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} s \\ -s-1 \\ 0 \end{pmatrix},\quad \overrightarrow{\mathrm{OQ}} = \begin{pmatrix} t \\ t \\ -2t+2 \end{pmatrix}$$

¢ ü PQ U 7 – q s ” t x , $\overrightarrow{\mathrm{PQ}}$ U Ú ¢ ℓ, m w f • g • w M ² Õ « Ä ç $\boldsymbol{v}_1, \boldsymbol{v}_2$ t 0 ` , ž t ( Ú q s ” q V p K ” . ` h U l o ,

$$\left(\overrightarrow{\mathrm{PQ}}, \boldsymbol{v}_1\right) = \left(\begin{pmatrix} t-s \\ t+s+1 \\ -2t+2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\right) = -2s-1 = 0 \Rightarrow s = -\frac{1}{2}$$

$$\left(\overrightarrow{\mathrm{PQ}}, \boldsymbol{v}_2\right) = \left(\begin{pmatrix} t-s \\ t+s+1 \\ -2t+2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right) = 6t-3 = 0 \Rightarrow t = \frac{1}{2}$$

\ w q V , $\left\|\overrightarrow{\mathrm{PQ}}\right\| = \sqrt{3}$. □

r q ` o , $\left\|\overrightarrow{\mathrm{PQ}}\right\|^2$ U 7 – q s ” q V , J ™ › ¬ h b .

$$\left\|\overrightarrow{\mathrm{PQ}}\right\|^2 = (t-s)^2 + (t+s+1)^2 + (-2t+2)^2 = 2\left(s+\frac{1}{2}\right)^2 + 6\left(t-\frac{1}{2}\right)^2 + 3$$

` h U l o , $s = -\dfrac{1}{2},\ t = \dfrac{1}{2}$ w q V 7 – ‹ $\sqrt{3}$ › q ” . □

« J 1.13 : A(1, −1, 0) › è “ , Ú ¢ ℓ : $x = y = z$ q ¦ : › Ë j , T m $\dfrac{\pi}{3}$ w s b ¯ › Ë m Ú ¢ w M Ü › { Š ‘ .

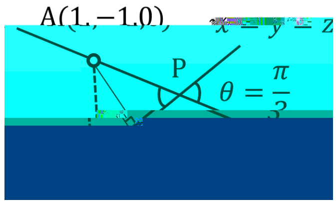


Fig. 1.10

¬ r t - Fig. 1.10 w ‘ O t G ø › ” . ‡ c , : A T ’ Ú ¢ ℓ t < ` h ( ¢ w H › { Š ” . : H x Ú ¢ $\ell$ Í t K ” w p , Í å Ý ” » $t$ › ‹ Ö ` , $x = y = z = t$ q S X \ q t ‘ “ ,

$$\overrightarrow{\mathrm{OH}} = \begin{pmatrix} t \\ t \\ t \end{pmatrix}$$

\ w q V , $\overrightarrow{\mathrm{AH}}$ q Ú ¢ w M ² Õ « Ä ç $\boldsymbol{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ U (Ú s w p ,

$$\left(\overrightarrow{\mathrm{AH}}, \boldsymbol{v}\right) = \left(\begin{pmatrix} t-1 \\ t+1 \\ t \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right) = 3t = 0 \Rightarrow t = 0$$

` h U l o , H(0, 0, 0). ‡ h , $\left\|\overrightarrow{\mathrm{AH}}\right\| = \sqrt{2}$. ^ ’ t , Fig. 1.10 ‘ “ , $\left\|\overrightarrow{\mathrm{HP}}\right\| = \dfrac{\sqrt{2}}{\sqrt{3}}$. Ž í ‘ “ , : P x ,

$$\overrightarrow{\mathrm{OP}} = \overrightarrow{\mathrm{OH}} + \overrightarrow{\mathrm{HP}} = \overrightarrow{\mathrm{HP}} = \pm\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \pm\frac{\sqrt{2}}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

h i ` , Fig. 1.10 t K ” ‘ O t , P' ‹ ß Q ’ • ” w p , ®± ¯ › ß Q o M ” .

74$t ,{Š"Ú¢x ,: A›è" ,M²Õ«ÄçU $\overrightarrow{\mathrm{AP}}$ wÚ¢pK" . bs ˜j ,

$$\frac{x-1}{\pm\sqrt{2}-3}=\frac{y+1}{\pm\sqrt{2}+3}=\frac{z}{\pm\sqrt{2}} \qquad \square$$

hi` ,óø‰qpK" .

«™1.11 rq`o ,2ªf t'"MO›ßQ" . : P›ÍåÝ"»¯Ôb•y ,

$\overrightarrow{\mathrm{OP}}=\begin{pmatrix}s\\s\\s\end{pmatrix}$ \wqV ,$\overrightarrow{\mathrm{AP}}$ qÚ¢wM²Õ«Äç $\boldsymbol{v}=\begin{pmatrix}1\\1\\1\end{pmatrix}$ U $\frac{\pi}{3}$ w¯›sbwp ,

$$\cos\frac{\pi}{3}=\frac{1}{2}=\frac{\left(\overrightarrow{\mathrm{AP}},\boldsymbol{v}\right)}{\left\|\overrightarrow{\mathrm{AP}}\right\|\|\boldsymbol{v}\|}=\frac{3s}{\sqrt{3s^2+2}\cdot\sqrt{3}} \Rightarrow s=\pm\frac{\sqrt{2}}{3}$$

`hUlo ,M²Õ«Äç

$$\overrightarrow{\mathrm{AP}}={}^t\left(\pm\frac{\sqrt{2}}{3}-1 \quad \pm\frac{\sqrt{2}}{3}+1 \quad \pm\frac{\sqrt{2}}{3}\right)=\frac{1}{3}\begin{pmatrix}\pm\sqrt{2}-3\\\pm\sqrt{2}+3\\\pm\sqrt{2}\end{pmatrix}$$

›˜" .

ð1.11 : A(2,1,0)›Ú¢ $\ell: x=y=z$ ›àq`o $\frac{\pi}{2}$ s8`h2ª›{Š' .

1.2.7 í tSZ" ØwM Ü

\w…px , ØwM Ü›‹Zb" . ‡c ,«™1.10 pb;`hí tSZ " ØwÍåÝ"»¯Ô›ºpb" .

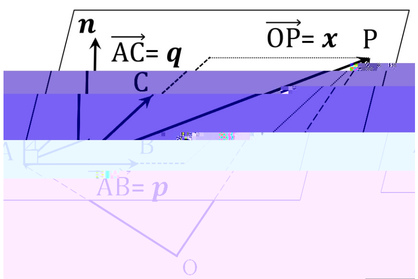


Fig. 1.11

[1.6 ¢ í tSZ” ØwÍåÝ”»¯Ô £ í tSZ” ØÍw:
P ›¯bÕ«Äç $\boldsymbol{x}$ x, ØÍwK” 1 :›¯bÕ«Äç $\boldsymbol{x}_0$ q ØÍtK
” 2 mw°Í qsÕ«Äç $\boldsymbol{p}, \boldsymbol{q}(\neq \boldsymbol{0})$ qÍåÝ”» s, t ›;Mo

$$\overrightarrow{\mathrm{OP}} = \overrightarrow{\mathrm{OA}} + s\overrightarrow{\mathrm{AB}} + t\overrightarrow{\mathrm{AC}}$$

$$\Leftrightarrow \boldsymbol{x} = \boldsymbol{x}_0 + s\boldsymbol{p} + t\boldsymbol{q} \tag{1.4}$$

q¯d”}\w‘Os¯Ô›í tSZ” ØwÍåÝ”»¯ÔqMO}

ØwÕ«ÄçM Ü $\boldsymbol{x} = \boldsymbol{x}_0 + s\boldsymbol{p} + t\boldsymbol{q}$ t

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \ \boldsymbol{x}_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

›EÖ`o , ^’t , $\boldsymbol{p}, \boldsymbol{q}$ w†Mt(ÚsÕ«Äç $\boldsymbol{n}$, bs˜j , $\boldsymbol{n} = \boldsymbol{p} \times \boldsymbol{q} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
›ß€b”q , $(\boldsymbol{n}, \boldsymbol{p}) = (\boldsymbol{n}, \boldsymbol{q}) = 0$ swp ,

$$(\boldsymbol{n}, \boldsymbol{x} - \boldsymbol{x}_0) = (\boldsymbol{n}, s\boldsymbol{p} + t\boldsymbol{q}) = 0$$

$$\Leftrightarrow \left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} \right) = a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

qs” . `hUlo , $d = -ax_0 - by_0 - cz_0$ qSZy

$$ax + by + cz + d = 0 \tag{1.5}$$

q¯d” . m‡“í ºw ØwM Üx $x, y,$ S‘| z w°ÍM Üqs” .
ŒwALT’, y0›tŽ<w [›\,”\qUZR” .

[1.7 ¢ í ºw ØwÕ«ÄçM Ü £ í tSZ” ØÍw:›¯
bÕ«Äç $\boldsymbol{x}$ x| ØÍwK” 1 :›¯bÕ«Äç $\boldsymbol{x}_0$ q Øq(ÚsÕ«
Äç $\boldsymbol{n}(\neq \boldsymbol{0})$ ›;Mo

$$(\boldsymbol{n}, \boldsymbol{x} - \boldsymbol{x}_0) = 0$$

q¯d"} $\boldsymbol{n}$ ›\w ØwO¢Õ«ÄçqMO . \wqV , ØÍw: (x_0, y_0, z_0), S'|O¢Õ«Äç $\boldsymbol{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ t0`o ,

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \tag{1.6}$$

qG\pV" .

ð 1.12 : A(2,1,1) ›è" , O¢Õ«ÄçU $\boldsymbol{n} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ pK" ØwM Ü›{Š' .

«J 1.14 3 : A(2,1,−1), B(1,0,1), C(−2,1,1) ›è" ØwM Ü›{Š' .

¬rt- ØwM Üt `o , ØÍwÚ¢w> ‰7t ([1.2 €°),

® Øx ,1 :qO¢Õ«Äçp>‡" . ¯

`hUlo , $\overrightarrow{AB}, \overrightarrow{AC}$ w†Mt(ÚsÕ«Äç $\boldsymbol{n}$ ›Žut'lo{Š ,(1.6) t 'lo , ØwM Ü›-‰b" . \wM t'•y ,

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$$

›{Š , Ít(Ú ›{Š•y'M . -‰› otb"hŠt ,$\overrightarrow{AC}$ U $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ t æpK"\q›b;`o ,{Š" ØwO¢Õ«Äç $\boldsymbol{n}$ x,

$$\boldsymbol{n} = \overrightarrow{AB} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

ŽÍ'" ,

$$1 \cdot (x - 2) + 3(y - 1) + 2(z + 1) = 0 \Leftrightarrow x + 3y + 2z = 3 \qquad \square$$

«™1.12 rq`o ,{Š" ØwM Ü›

$$ax + by + cz + d = 0$$

q S V , 3 : A(2,1,−1), B(1,0,1), C(−2,1,1) › E Ö b • y ,

$$\begin{cases} 2a+b-c+d=0 & \text{(i)} \\ a+c+d=0 & \text{(ii)} \\ -2a+b+c+d=0 & \text{(iii)} \end{cases}$$

› ¬ b ” . \ • x , 4 i È q ° Í M Ü s w p , $a:b:c:d$ › { Š ” \ q U è ª q s ” « 4. (i)+(iii) ‘ “ , $b+d=0$. (i) t E Ö ` o , $2a-c=0$. (ii) ‘ “ , $3a+d=0$. Ž Í ‘ “ ,

$$a:b:c:d = a:3a:2a:-3a = 1:3:2:-3 \quad a\neq 0$$

Ž Í ‘ “ $x+3y+2z=3$ › ˜ ” . \ w M O x , ° j › $ p K ” U , M Ü › r X ž A Q T ’ Ì U T T ” \ q U É : p K ” .

ð 1.13 3 : A(0,1,−1), B(1,2,1), C(2,1,0) › è ” Ø w M Ü › { Š ‘ .

ð 1.14 : A(1,1,0) › è “ , Ú ¢ $x=2(y-1)=z$ › ‰ Ø w M Ü › { Š ‘ .

ð 1.15 2 Ú ¢ $x=y=z-1$, $x+1=y-2=z-2$ w † M › ‰ Ø w M Ü › { Š ‘ .

ð 1.16 2 Ú ¢ $x-1=\dfrac{y}{2}=\dfrac{z-1}{3}$, $\dfrac{x+1}{5}=\dfrac{y}{2}=z-2$ U 1 : p ¦ ˜ ” \ q › Ô ` , f w † M › ‰ Ø w M Ü › { Š ‘ .

« J 1.15 2 Ø $x-y+z=1$, $2x+y-z=2$ w ¦ ¢ w M Ü › { Š ‘ .

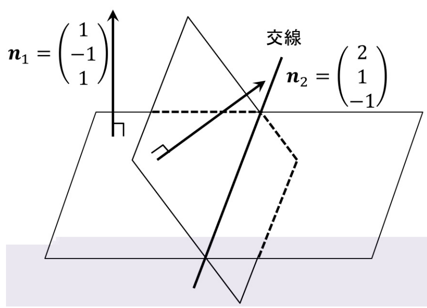


Fig. 1.12

¬ r t - ‡ c , ¦ ¢ › è ” : › { Š ” ž A U K ” . q “ K Q c $z=0$ q b • y , $x-y=1$, $2x+y=2$ q s “ , \ • › r Z y , (1,0,0) › ˜ ” . Í t , M ² Õ « Ä ç

« 4 î M , ¢ E : ¶ I p Ä I › † Ì b ”

x, ØwO¢Õ«Äç $\boldsymbol{n}_1 = \begin{pmatrix}1\\-1\\1\end{pmatrix}, \boldsymbol{n}_2 = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ w†Mt(ÚpK” . `hUI

o, Žut‘“ , $\boldsymbol{v} = \begin{pmatrix}0\\1\\1\end{pmatrix}$ ›˜” . Ží‘“ ,

$$x=1,\ y=z$$ □

«™1.13 $z=0$ pq“KQc¦¢Íw:›{ŠhU , Ôùt‘lox , $y=0$ • $z=0$ q `o‹‘M .

$x-y+z=1, 2x+y-z=2$ T’ 2 Ü› `o , $x=1$. \•‘“ , Øtí`o , $-y+z=0$ ›˜” . \•x , Ú¢›¯b .

ð 1.17 2 Ø $x+y+z=0, 2x+y-z=2$ w¦¢wM Ü›{Š‘ .

> [1.8 í ºw: A(x_0,y_0,z_0) T’ Ø $\alpha : ax+by+cz+d=0$ t<– `h(¢w › H qSXqV|¢ü AH wÕ^›: A q Ø α w‘mq [b” .

\wqV , : A(x_0,y_0,z_0) T’ Ø $\alpha : ax+by+cz+d=0$ w‘m L x

$$L=\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$

p)Q’•” .

¬ ÂÌ- (¢w H(x_1,y_1,z_1) x, : A(x_0,y_0,z_0) è“ , M²Õ«Äç $\begin{pmatrix}a\\b\\c\end{pmatrix}$ t

ËmÚ¢ÍtK”wp ,

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix}+t\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}x_0+at\\y_0+bt\\z_0+ct\end{pmatrix}$$

°M, : H x, Ø α ÍtK”wp , EÖ`o ,

$$a(x_0+at)+b(y_0+bt)+c(z_0+ct)+d=0$$

$$\Rightarrow t=t_1=-\frac{ax_0+by_0+cz_0+d}{a^2+b^2+c^2}$$

` h U l o ,

$$\begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{pmatrix} = t_1 \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\frac{ax_0 + by_0 + cz_0 + d}{a^2 + b^2 + c^2} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\Rightarrow L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \qquad \square$$

1.2.8 í t S Z ” • Ø w M Ü

í º w • Ø x , Ø Í w q ‰ 7 t , ¤ ú q R p ‡ ” .

« J 1.16 ¤ ú $(a, b.c)$, R $r(>0)$ w • Ø M Ü x

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

p K ” . î M , \ w ‘ O s • Ø Í w : (x, y, z) x Í å Ý ” » θ, ϕ, $(0 \le \theta \le \pi$, $0 \le \phi < 2\pi)$ › ; M o

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{pmatrix}$$

q ¯ d ” . \ • ‘ “ , $x = a + r\sin\theta\cos\phi$, $y = b + r\sin\theta\sin\phi$, $z = c + r\cos\theta$ p K ” T ’ , θ, ϕ › « ˆ b • y • Ø w M Ü U ˜ ’ • ” .

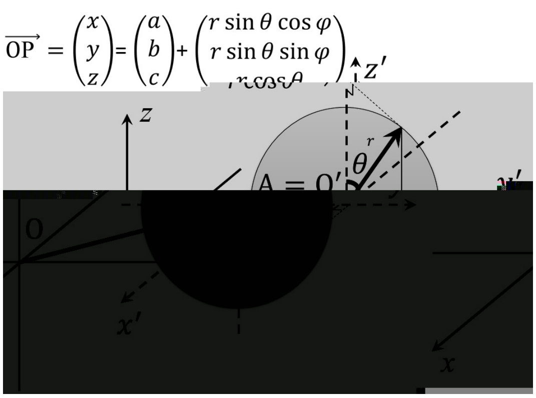


Fig. 1.13

« J 1.17 • Ø $(x-1)^2+(y+1)^2+z^2=6$ q Ú ¢ $x-1=\dfrac{y-2}{-2}=\dfrac{z-3}{-1}$ w ¦ : › { Š ‘ .

¬ r t - ‡ c , Ú ¢ Í w : P › Í å Ý ” » ¯ Ô b • y ,

$$\overrightarrow{\mathrm{OP}}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}1\\-2\\-1\end{pmatrix}=\begin{pmatrix}t+1\\-2t+2\\-t+3\end{pmatrix}$$

: P U • Ø Í t K ” w p E Ö ` o ,

$$t^2+(-2t+3)^2+(-t+3)^2=6 \Leftrightarrow t^2-3t+2=(t-1)(t-2)=0 \Leftrightarrow t=1,\ 2$$

` h U l o , $t=1$ w q V (2,0,2), $t=2$ w q V , (3,−2,1). □

ð 1.18 • Ø $x^2+y^2+(z-1)^2=11$ q Ú ¢ $x-2=\dfrac{y-3}{2}=\dfrac{z-2}{-2}$ w ¦ : › { Š ‘ .

« J 1.18 • Ø $x^2+y^2+z^2-4x+2y-6z-11=0$ q Ø $x-2y+2z=1$ U K ” . • Ø q Ø w ¦ ˜ “ w w ¤ ú q R › { Š ‘ .

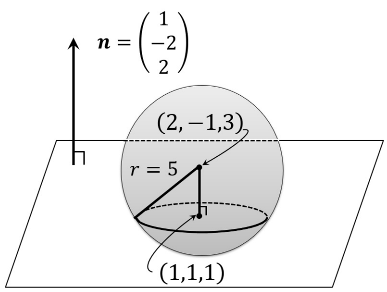


Fig. 1.14

¬ r t - $(x-2)^2+(y+1)^2+(z-3)^2=5^2$ q ! p V ” w p , ¤ ú U (2,−1,3), R $r=5$ w • Ø › ¯ b . w ¤ ú w 2 ª x , ¤ ú (2,−1,3) › è “ , Ø w O ¢ Õ « Ä ç › M ² Õ « Ä ç $\begin{pmatrix}1\\-2\\2\end{pmatrix}$ t ‹ m Ú ¢ q Ø w ¦ : p K ” w p , w ¤ ú

› Í å Ý ” » ¯ Ô b • y ,

$$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} t+2 \\ -2t-1 \\ 2t+3 \end{pmatrix}$$

\ w : U Ø Í t K ” w p E Ö ` o ,

$$t+2-2(-2t-1)+2(2t+3)=1 \Leftrightarrow t=-1$$

` h U l o , w ¤ ú x , (1,1,1). ‡ h , ¤ ú (2,−1,3) T ’ Ø $x-2y+2z=1$ ‡ p w ‘ m L x ,

$$L = \frac{|2+(-2)\cdot(-1)+6-1|}{\sqrt{1^2+(-2)^2+2^2}} = 3$$

Ž Í T ’ , w R R x , $R=\sqrt{5^2-3^2}=4$. □

« ™ 1.14 • Ø w ¤ ú (2,−1,3) q (1,1,1) w ¤ ú q w ‘ m q ` o ,

$$L=\sqrt{(2-1)^2+(-1-1)^2+(3-1)^2}=3$$

q ` o - ‰ ` o ‹ ‘ M .

ð 1.19 • Ø $x^2+y^2+z^2=1$ q Ø $x+y+z=1$ U K ” . • Ø q Ø w ¦ ˜ “ w w ¤ ú q R › { Š ‘ .

> « J 1.19 4 : A(2,2,0), B(2,0,4), C(−1,2,1), D(3,2,3) › è ” • Ø w M Ü › { Š ‘ .

¬ r t - { Š ” • Ø w M Ü ›

$$x^2+y^2+z^2+ax+by+cz+d=0$$

q S X . 4 : › è ” w p E Ö b • y ,

$$\begin{cases} 2a+2b+d=-8 \\ 2a+4c+d=-20 \\ -a+2b+c+d=-6 \\ 3a+2b+3c+d=-22 \end{cases} \tag{1.7}$$

q s ” . \ • ’ › r M o , $(a,b,c,d)=(-2,-2,-4,0)$. Ž Í ‘ “ ,

$$x^2+y^2+z^2-2x-2y-4z=0$$ □

ð 1.20 4 : A(−2,8,−1), B(−2,5,6), C(0,−2,1), D(3,3,3) › è ” • Ø w M Ü › { Š ‘ .

è × , ô s ¶ Í t S M o , 4 i È q ° Í M Ü (1.7) › r X \ q x , ; • $ s - ‰ t s ” w p , ^ æ ^ • ” \ q x Š l h t s M . ` T ` s U ’ , ¢ E : ¶ p x , o i q ` o ® È q ° Í M Ü ¯ U O b ” . î M , ¦ ï Ã Ú ï Å è [ p x , \ w o i › ‰ . \ \ p x , È q ° Í M Ü › ¯ ï Ð á ” » t ‘ l o { Š ” \ q › > ` o , ß V Z ` O (sweep-out method/row reduction) « [5] t m M o † Ì b ” .

‡ c , Ž < w o s 3 i È q ° Í M Ü › ß V Z ` O t ‘ l o r X .

$$\begin{cases} x+y-2z=3 & \text{(i)} \\ -x+y+z=0 & \text{(ii)} \\ 2x-y-z=3 & \text{(iii)} \end{cases}$$

ß V Z ` O w , Š x ,

® M Ü w : › w M Ü t C Q o ‹ ! ˜ ’ s M ¯

Q í › b ; b ” . b s ˜ j , (ii)+(i) • , (iii)-(i) ×2 s w â ^ › æ O .

$$\begin{cases} & x+y-2z=3 & \text{(i)} \\ \text{(ii)}+\text{(i)} & 0x+2y-z=3 & \text{(ii)}' \\ \text{(iii)}-\text{(i)}\times 2 & 0x-3y+3z=-3 & \text{(iii)}' \end{cases}$$

Í t , (ii)′ › ¶ . 2 p Â l o , Ž < w â ^ › æ s O .

$$\begin{cases} \text{(i)}-\text{(ii)}'' & x+0y-\frac{3}{2}z=\frac{3}{2} & \text{(i)}'' \\ \text{(ii)}'\times\frac{1}{2} & 0x+y-\frac{1}{2}z=\frac{3}{2} & \text{(ii)}'' \\ \text{(iii)}'+\text{(ii)}''\times 3 & 0x+0y+\frac{3}{2}z=\frac{3}{2} & \text{(iii)}'' \end{cases}$$

¾ V V , (iii)″ › ¶ . $\frac{3}{2}$ p Â l o , Ž < w â ^ › æ s O .

$$\begin{cases} \text{(i)}''+\text{(iii)}'''\times\frac{3}{2} & x+0y+0z=3 & \text{(i)}''' \\ \text{(ii)}''+\text{(iii)}'''\times\frac{1}{2} & 0x+y+0z=2 & \text{(ii)}''' \\ \text{(iii)}''\times\frac{2}{3} & 0x+0y+z=1 & \text{(iii)}''' \end{cases}$$

Ž Í ‘ “ , $(x,y,z)=(3,2,1)$ › ˜ ” . \ • › Ü $ t Ž < w ‘ O t G \ b ” .

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ -1 & 1 & 1 & 0 \\ 2 & -1 & -1 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 2 & -1 & 3 \\ 0 & -3 & 3 & -3 \end{array}\right]$$

[5] ¨ ¢ µ w « ˆ O (Gaussian elimination) q ‹ z y • ” .

$$
\to \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & \frac{3}{2} & \frac{3}{2} \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]
$$

\ \ p , ° j È U r › ¯ ` o M " .

7 ™ t , ß V Z ` O t ' l o , (1.7) › r X .

$$
\left[\begin{array}{cccc|c} 2 & 2 & 0 & 1 & -8 \\ 2 & 0 & 4 & 1 & -20 \\ -1 & 2 & 1 & 1 & -6 \\ 3 & 2 & 3 & 1 & -22 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 0 & \frac{1}{2} & -4 \\ 0 & -2 & 4 & 0 & -12 \\ 0 & 3 & 1 & \frac{3}{2} & -10 \\ 0 & -1 & 3 & -\frac{1}{2} & -10 \end{array}\right]
$$

$$
\to \left[\begin{array}{cccc|c} 1 & 0 & 2 & \frac{1}{2} & -10 \\ 0 & 1 & -2 & 0 & 6 \\ 0 & 0 & 7 & \frac{3}{2} & -28 \\ 0 & 0 & 1 & -\frac{1}{2} & -4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 2 & \frac{1}{2} & -10 \\ 0 & 1 & -2 & 0 & 6 \\ 0 & 0 & 1 & \frac{3}{14} & -4 \\ 0 & 0 & 1 & -\frac{1}{2} & -4 \end{array}\right]
$$

$$
\to \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{14} & -2 \\ 0 & 1 & 0 & \frac{3}{7} & -2 \\ 0 & 0 & 1 & \frac{3}{14} & -4 \\ 0 & 0 & 0 & -\frac{5}{7} & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]
$$

\ \ p , ° j È t K " $(a,b,c,d) = (-2,-2,-4,0)$ U r › ¯ ` o M " . □

« ™1.15 È q ° Í M Ü › ß Q " Ô ù , ° Œ : U M Ü w : ' " M Ô ù U O b " . « Q y , « J 1.15 p x , $x-y+z=1, 2x+y-z=2$ › 2 Ø q r ` h U , \ • › ° Œ : U $x, y, z$ w 3 m t 0 ` o , M Ü w : U 2 m p K " È q ° Í M Ü q ß Q " \ q U p V " . — ° ` = ` o , Ž < w ð J › ß Q " .

a › : q b " . È q ° Í M Ü $x-y+z=1, 2x+y+az=2$ › r Z .

ß V Z ` O t ' l o r X . › $ t ¯ q b • y ,

$$
\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 1 & a & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & a-2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & \frac{a+1}{3} & 1 \\ 0 & 1 & \frac{a-2}{3} & 0 \end{array}\right]
$$

\ • x ,

$$
\begin{cases} x + \dfrac{a+1}{3} z = 1 \\ y + \dfrac{a-2}{3} z = 0 \end{cases}
$$

q ‰ ‹ p K " . ` h U l o ,

(i) $a=-1$ w q V , $x=1, y=z.$ \ • x , « J 1.15 q ‰ a A L p K ” .
(ii) $a=2$ w q V , $x+z=1, y=0.$
(ii) $a\neq-1$ T m $a\neq2$ w q V ,

$$\frac{x-1}{-a-1}=\frac{y}{-a+2}=\frac{z}{3} \tag{1.8}$$

q s ” .

(i), (ii), (iii) M c • ‹ 2 ? ¶ $ t _ • y Ú ¢ › ¯ b . ° M , È q ° Í M Ü q ß Q • y , ° Œ : U M Ô ù , z › Í å Ý ” » q ` o , r $x, y$ U Í å Ý ” » ¯ q ^ • ” \ q › ™ ¯ b ” . \ • ’ ‹ ¦ ï Ã Ú ï Å è [ p { O . j s ˆ t , Ž u › b ; b • y , 2 Ø w O ¢ Õ « Ä ç $\boldsymbol{n}_1=\begin{pmatrix}1\\-1\\1\end{pmatrix}, \boldsymbol{n}_2=\begin{pmatrix}2\\1\\a\end{pmatrix}$ w † M t ( Ú p K ” Õ « Ä ç w ° m q ` o , $\boldsymbol{v}=\begin{pmatrix}-a-1\\-a+2\\3\end{pmatrix}$ › ˜ ” . \ • x , (1.8) p ¯ ^ • ” Ú ¢ w M Ü t S Z ” M ² Õ « Ä ç q ° • ` o M ” .

> [1.9 ¢ • Ø w Õ « Ä ç M Ü £ í º t S Z ” • Ø Í w : › ¯ b Õ « Ä ç $\boldsymbol{x}$ x , ¤ ú w • ” Õ « Ä ç $\boldsymbol{a}$ q R r › ; M o
>
> $$\|\boldsymbol{x}-\boldsymbol{a}\|=r$$
>
> q ¯ d ” } \ w ‘ O s ¯ Ô › • Ø w Õ « Ä ç M Ü q M O }

> « J 1.20 xyz í t S M o , $\left\|\overrightarrow{\mathrm{OA}}\right\|=1,$
>
> $$\left(\overrightarrow{\mathrm{OP}},\overrightarrow{\mathrm{OA}}\right)^2+\left\|\overrightarrow{\mathrm{OP}}-\left(\overrightarrow{\mathrm{OP}},\overrightarrow{\mathrm{OA}}\right)\overrightarrow{\mathrm{OA}}\right\|^2=1$$
>
> › ¬ h b q V , : P w J { › { Š ‘ . (2009 • G ¶ (~ J))

¬ r t - $\overrightarrow{\mathrm{OP}}=\boldsymbol{p}, \overrightarrow{\mathrm{OA}}=\boldsymbol{a}$ q S X . h i ` , $\|\boldsymbol{a}\|=1$ p K ” . \ w q V ,

$$\begin{aligned}&(\boldsymbol{p},\boldsymbol{a})^2+\|\boldsymbol{p}-(\boldsymbol{p},\boldsymbol{a})\boldsymbol{a}\|^2=1\\ \Rightarrow\ &(\boldsymbol{p},\boldsymbol{a})^2+\|\boldsymbol{p}\|^2-2(\boldsymbol{p},\boldsymbol{a})^2+(\boldsymbol{p},\boldsymbol{a})^2\|\boldsymbol{a}\|^2=1\\ \Rightarrow\ &\|\boldsymbol{p}\|^2=1 \Leftrightarrow \|\boldsymbol{p}\|=1\end{aligned}$$

` h U l o , j : ¤ ú R 1 w • Ø › ¯ b . □

ð 1.21 1 % w Õ ^ U 2 p K ” Y › Ø . ABCD t º € b ” • Ø Í t : P › q ” . \ w q V, Ž < w ‹ U ° p K ” \ q › Ô d .

$$L=\left|\overrightarrow{\mathrm{AP}}\right|^2+\left|\overrightarrow{\mathrm{BP}}\right|^2+\left|\overrightarrow{\mathrm{CP}}\right|^2+\left|\overrightarrow{\mathrm{DP}}\right|^2$$

1.2.9 í tSZ” wÍåÝ”»¯Ô

•Øq Øw¦˜“x qs”U , fwÍåÝ”»¯Ôx , í tSZ” Ø wÍåÝ”»¯Ô (1.4) ›Mv`h Üp¯qpV” . é.$t ,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + r\cos\theta\, \boldsymbol{p} + r\sin\theta\, \boldsymbol{q}, \ \ \|\boldsymbol{p}\| = \|\boldsymbol{q}\| = 1, \ (\boldsymbol{p}, \boldsymbol{q}) = 0, \ 0 \leq \theta < 2\pi \quad (1.9)$$

pK•y , í p¤úU (x_0, y_0, z_0), R r w *›¯b . \\p , ØwÍå Ý”»¯Ô (1.4) tSMo , $s = r\cos\theta$, $t = r\sin\theta$ ‘“ , $s^2 + t^2 = 1$ qMOMvU C˜” .

> «J 1.21 í ºw: P (x, y, z) U $x^2 + y^2 + z^2 = 1$, $x + y + z = 0$ ›¬hbq V, : A(1,0,0) ‡pw‘m AP w7–‹›{Š‘ .

¬rt- \w¦˜“w$ x , Ì’Ttj:¤ú , R 1 w *pK” . ‡h, *Íw: P x, (1.9) t‘lo , ÍåÝ”» $0 \leq \theta < 2\pi$ ›b;`o , Ž<w‘O t¯qpV” .

$$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \cos\theta \, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \sin\theta \, \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$= \frac{\cos\theta}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \frac{\sin\theta}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

hi` , (1.7) wÕ«Äç $\boldsymbol{p}, \boldsymbol{q}$ w¬|Mx , ‡c , 0¶Q›ßQo , ‹lq‹ os ‹wp , $\boldsymbol{p} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ U¬RpV” . Ít , ØwO¢Õ«Äç $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ q $\boldsymbol{p}$ w†M t(Úso•Õ«Äçq`o , Žu›b;`o , $\boldsymbol{q} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ ›¬R`oM” .

` h U l o ,

$$\overrightarrow{\mathrm{AP}} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \frac{\cos\theta}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \frac{\sin\theta}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

s w p ,

$$\begin{aligned} |\overrightarrow{\mathrm{AP}}|^2 &= 1 + \cos^2\theta + \sin^2\theta - \frac{2}{\sqrt{2}}\cos\theta - \frac{2}{\sqrt{6}}\sin\theta \\ &= 2 - 2\sqrt{\frac{2}{3}}\sin\left(\theta + \frac{1}{3}\pi\right) \end{aligned}$$

q s “ , 7 – ‹ x , $\theta = \dfrac{\pi}{6}$ w q V $\sqrt{2 - 2\sqrt{\dfrac{2}{3}}}$ p K ” . □

1.3 n Í i Õ « Ä ç

1.3.1 n Í i Õ « Ä ç w , Å

n x w î : $a_1, a_2, \ldots, a_n$ w Ê › N t , o { M h

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n$$

› n Í i Õ « Ä ç q ‘ • . 2 m w n Í i Õ « Ä ç

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

q î : k t 0 ` o , è q î : ›

$$\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}, \ k\boldsymbol{a} = \begin{pmatrix} ka_1 \\ ka_2 \\ \vdots \\ ka_n \end{pmatrix}$$

¬ Â Ì - (2) w ˆ Â Ì › æ O .

$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

q S Z y , Ú ™ w î : t t 0 ` o ,

$$\sum_{k=1}^{n} (a_k t + b_k)^2 = \left(\sum_{k=1}^{n} a_k^2\right) t^2 + 2\left(\sum_{k=1}^{n} a_k b_k\right) t + \left(\sum_{k=1}^{n} b_k^2\right)$$
$$= \|\boldsymbol{a}\|^2 t^2 + (\boldsymbol{a}, \boldsymbol{b}) t + \|\boldsymbol{b}\|^2 \geq 0$$

\ \ p , $\|\boldsymbol{a}\| = 0$ w q V x , $\boldsymbol{a} = \boldsymbol{0}$ p R q b ” . ° M , $\|\boldsymbol{a}\| \neq 0$ w q V x , t t b ” 2 Í Æ s Ü q ß Q ” q , Q Ü x 0 Ž < p s Z • y s ’ s M .

$$\frac{D}{4} = (\boldsymbol{a}, \boldsymbol{b})^2 - \|\boldsymbol{a}\|^2 \cdot \|\boldsymbol{b}\|^2 \leq 0$$
$$\Rightarrow \left|(\boldsymbol{a}, \boldsymbol{b})\right| \leq \left\|\boldsymbol{a}\right\| \left\|\boldsymbol{b}\right\| \qquad \square$$

‡ h , s ø x , $\boldsymbol{a} = k\boldsymbol{b}$ w q V R q b ” .

« J 1.22 Ã ” » B ù $(x_k, y_k), (k = 1, 2, \ldots, n)$ t 0 ` o , ì : (correlation coef cient) x , Ž < w ‘ O t [^ • ” .

$$\rho_{xy} = \frac{\mathrm{Cov}(X,\ Y)}{\sqrt{V(X)}\sqrt{V(Y)}} = \frac{\mathrm{Cov}(X,\ Y)}{\sigma_x \sigma_y}$$

h i ` , ü „ (variance) , ž ü „ (covariance) x , Ž < w ‘ O t [^ • ” .

$$V(X) = \sigma_x^2 = E\big((X - \mu_x)^2\big) = \frac{1}{n}\Big((x_1 - \mu_x)^2 + (x_2 - \mu_x)^2 + \cdots + (x_n - \mu_x)^2\Big)$$
$$= \frac{1}{n}\sum_{k=1}^{n} (x_k - \mu_x)^2 = E(X^2) - \mu_x^2$$
$$V(Y) = \sigma_y^2 = E\big((Y - \mu_y)^2\big) = \frac{1}{n}\Big((y_1 - \mu_y)^2 + (y_2 - \mu_y)^2 + \cdots + (y_n - \mu_y)^2\Big)$$
$$= \frac{1}{n}\sum_{k=1}^{n} (y_k - \mu_y)^2 = E(Y^2) - \mu_y^2$$
$$\mathrm{Cov}(X, Y) = E((X - \mu_x)(Y - \mu_y))$$
$$= \frac{1}{n}\sum_{k=1}^{n} \big(x_k - \mu_x\big)\big(y_k - \mu_y\big) = E(XY) - \mu_x \mu_y$$

$$E(XY) = \frac{x_1 y_1 + x_2 y_2 + \cdots + x_n y_n}{n} = \frac{1}{n} \sum_{k=1}^{n} x_k y_k$$

$$\mu_x = E(X) = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n} \sum_{k=1}^{n} x_k$$

$$\mu_y = E(Y) = \frac{y_1 + y_2 + \cdots + y_n}{n} = \frac{1}{n} {}_{1}\; x_k$$

h i ` ,

$$
\begin{aligned}
s_{xy} &= \frac{1}{n}\Big((x_1-\overline{x})(y_1-\overline{y})+(x_2-\overline{x})(y_2-\overline{y})+\cdots+(x_n-\overline{x})(y_n-\overline{y})\Big)\\
&= \frac{1}{n}\sum_{k=1}^{n}(x_k-\overline{x})(y_k-\overline{y}) = \frac{1}{n}\sum_{k=1}^{n}x_k y_k - \overline{x}\,\overline{y}
\end{aligned}
$$

$$
s_x^2 = \frac{1}{n}\Big((x_1-\overline{x})^2+(x_2-\overline{x})^2+\cdots+(x_n-\overline{x})^2\Big) = \frac{1}{n}\sum_{k=1}^{n}(x_k-\overline{x})^2 = \frac{1}{n}\sum_{k=1}^{n}x_k^2-\overline{x}^2
$$

$$
s_y^2 = \frac{1}{n}\Big((y_1-\overline{y})^2+(y_2-\overline{y})^2+\cdots+(y_n-\overline{y})^2\Big) = \frac{1}{n}\sum_{k=1}^{n}(y_k-\overline{y})^2 = \frac{1}{n}\sum_{k=1}^{n}y_k^2-\overline{y}^2
$$

$$
\overline{x} = \frac{1}{n}\sum_{k=1}^{n}x_k,\ \overline{y} = \frac{1}{n}\sum_{k=1}^{n}y_k
$$

« J 1.23 : $d \neq 0, a_1, a_2, \ldots, a_n$ ›) Q h q V ,

$$
a_1x_1 + a_2x_2 + \cdots + a_nx_n + d = 0
$$

› ¬ h ` s U ' ˆ X q b " . \ w q V , :

$$
f = x_1^2 + x_2^2 + \cdots + x_n^2
$$

w 7 – ‹ › { Š ' .

¬ r t - ‡ c ,

$$
\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix},\ \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}
$$

q b • y , ³ á ë ç À w Æ s Ü t ' " ,

$$
\begin{aligned}
(-d)^2 &= (a_1x_1 + a_2x_2 + \cdots + a_nx_n)^2 = (\boldsymbol{a}, \boldsymbol{x})^2\\
&\leq \|\boldsymbol{a}\|^2\|\boldsymbol{x}\|^2 = (a_1^2 + a_2^2 + \cdots + a_n^2)(x_1^2 + x_2^2 + \cdots + x_n^2)
\end{aligned}
$$

` h U l o ,

$$
f = x_1^2 + x_2^2 + \cdots + x_n^2 \geq \frac{d^2}{a_1^2 + a_2^2 + \cdots + a_n^2} = \frac{d^2}{\|\boldsymbol{a}\|^2} \qquad \square
$$

h i ` , s ø x ,

$$
x_k = x_k^* = -\frac{d\,a_k}{\|\boldsymbol{a}\|^2},\ k = 1, 2, \ldots, n
$$

w q V t R “ q m .

« ™ 1.17 $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n, \boldsymbol{a} \neq 0$ t 0 ` o ,

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n + d = 0$$

› ¬ h b $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$ w B ù › Ò Ø (hyperplane) q M O. \ • x , í t S Z ”
Ø w M Ü (1.5) w ° ` = p K ” .

3 Í i í q ‰ 7 t , n Í i Õ « Ä ç í º w : A$(\alpha_1, \alpha_2, \ldots, \alpha_n)$ T ’ Ò Ø

$$\Gamma : a_1x_1 + a_2x_2 + \cdots + a_nx_n + d = 0$$

t < – ` h (¢ w › H q S X q V | ¢ ü AH w Õ ^ › : A q Ø Γ w ‘ m q [b ” . \ w q V ,

$$L = \frac{|a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n + d|}{\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}} = \frac{\left|\sum_{k=1}^{n} a_k\alpha_k + d\right|}{\sqrt{\sum_{k=1}^{n} a_k^2}}$$

p) Q ’ • ” . \ w A L › b ; b • y , f w 7 – ‹ x , ‘ m AH w 2 Ð t s ` M . b s ˜ j , Ž < w ‘ O t - ‰ ^ • ” .

$$f = x_1^2 + x_2^2 + \cdots + x_n^2 \geq \frac{|d|^2}{\sqrt{\sum_{k=1}^{n} a_k^2}^{\,2}} = \frac{d^2}{\|\boldsymbol{a}\|^2}$$

□

« ™ 1.18 ¶ X Ÿ s ” M O q ` o , å ¬ å ï ´ á w ° Ð : O (method of Lagrange multiplier) t ‘ ” r O U K ” ¦ 2 §. b s ˜ j , $a_1x_1 + a_2x_2 + \cdots + a_nx_n + d = 0$ › Æ Ú E q ` o , f U 7 – q s ” Ú E › ‹ Z b ” .

‡ c , å ¬ å ï ´ á :

$$\begin{aligned} \mathscr{L} &= x_1^2 + x_2^2 + \cdots + x_n^2 + L(a_1x_1 + a_2x_2 + \cdots + a_nx_n + d) \\ &= \sum_{k=1}^{n} x_k^2 + L\left(\sum_{k=1}^{n} a_kx_k + d\right) \end{aligned}$$

› [b ” . h i ` , L x å ¬ å ï ´ á : (Lagrange multiplier) p K ” . \ w q V ,
• ü (partial derivative) › æ s M

$$\frac{\partial \mathscr{L}}{\partial x_k} = 2x_k + La_k = 0 \Rightarrow x_k = x_k^* = -\frac{L^*}{2} a_k, \ k = 1, 2, \ldots, n$$

› ˜ ” « [7]. Í t , Æ Ú E t E Ö ` ,

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + d = -\frac{L}{2} \sum_{k=1}^{n} a_k^2 + d = 0$$

$$\Rightarrow L = L^* = \frac{2d}{\sum_{k=1}^{n} a_k^2}$$

› ˜ ” . ` h U l o ,

$$x_k = x_k^* = -\frac{L^*}{2} a_k = -\frac{d a_k}{\sum_{k=1}^{n} a_k^2}, \ k = 1, 2, \ldots, n$$

Ž Í ‘ “ ,

$$f \geq \sum_{k=1}^{n} (x_k^*)^2 = \frac{d^2 (a_1^2 + a_2^2 + \cdots + a_n^2)}{\left(\sum_{k=1}^{n} a_k^2 \right)^2} = \frac{d^2}{\|\boldsymbol{a}\|^2} \qquad \square$$

h i ` , ž A Ú E t a W s M \ q t « ™ U ž A p K ” .

« [7] $*$ x 7 & Q w ™ ¯ p K ” . b s ˜ j , 7 – ‹ ›) Q ” ! : p K ” .

H 1 ·

ð 1.1 $\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2-t \\ 1+4t \end{pmatrix}$.. ƒ M x , $\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1+t \\ 5-4t \end{pmatrix}$. ‹ D .

ð 1.2 ‡ c , ˆ : P w Í å Ý ” » ¯ Ô x , $\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ 2-t \end{pmatrix}$ p K ” .

` h U l o , $\left(\overrightarrow{OP}, \overrightarrow{OA}\right) = 2t + 2(2-t) = 4$ p ° p K ” .

ð 1.3 $\boldsymbol{h} = \frac{1}{6}\boldsymbol{a} + \frac{5}{9}\boldsymbol{b}$

ð 1.4 $\|\boldsymbol{x}\|^2 - 2(\boldsymbol{a}, \boldsymbol{x}) = 0 \Rightarrow \|\boldsymbol{x} - \boldsymbol{a}\|^2 = \|\boldsymbol{a}\|^2 \Leftrightarrow \|\boldsymbol{x} - \boldsymbol{a}\| = \|\boldsymbol{a}\|$ q ! p V ” w p , ¤ ú U $\boldsymbol{a}$, R $r = \|\boldsymbol{a}\|$ w › ¯ b . s S , \ w x j : › è ” .

ð 1.5 $\frac{\pi}{4}$. [$\cos\angle BAP = \frac{\left(\overrightarrow{AP}, \overrightarrow{AB}\right)}{\|\overrightarrow{AP}\|\|\overrightarrow{AB}\|} = \frac{2(t^2+2)}{\sqrt{t^4+4t^2+4}\cdot 2\sqrt{2}} = \frac{1}{\sqrt{2}}$]

ð 1.6 $S = \frac{1}{2}\sqrt{4a^2 + b^2 + 4}$.

ð 1.7 $\boldsymbol{e} = \pm\frac{1}{\sqrt{14}}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

ð

$\boldsymbol{x}$

ðövO %øsPÑBÐF“Ú—d ñ73%÷$ Ó ñW3%öd wÓ ñ #2_ i=_g3%÷ jöb_3%ß s2_A ö—3%ô 7mñD•mÀF“Õ¯g3%õ $)0ogwOwfA'}0? r3%ðF“Öös2_p &Ó b3%Õ

ð 1.18 $(1,1,4)$, $\left(\frac{5}{3},\frac{7}{3},\frac{8}{3}\right)$.

ð 1.19 ¤ ú $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$, R $\frac{\sqrt{6}}{3}$.

ð 1.20 $x^2+y^2+z^2+4x-6y-2z-15=0$.

ð 1.21 $L=|\overrightarrow{\mathrm{AP}}|^2+|\overrightarrow{\mathrm{BP}}|^2+|\overrightarrow{\mathrm{CP}}|^2+|\overrightarrow{\mathrm{DP}}|^2=\dfrac{20}{3}$. [º € • w ¤ ú › O q b ” . ‡ h , $\overrightarrow{\mathrm{OP}}=\boldsymbol{p}$, $\overrightarrow{\mathrm{OA}}=\boldsymbol{a}$, $\overrightarrow{\mathrm{OB}}=\boldsymbol{b}$, $\overrightarrow{\mathrm{OC}}=\boldsymbol{c}$, $\overrightarrow{\mathrm{OD}}=\boldsymbol{d}$ q S X . ¤ ú O x Y › Ø . w O ú s w p , $\frac{1}{4}(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}+\boldsymbol{d})=\boldsymbol{0}$ t « ™ ` o , $L=4\|\boldsymbol{p}\|^2+\|\boldsymbol{a}\|^2+\|\boldsymbol{b}\|^2+\|\boldsymbol{c}\|^2+\|\boldsymbol{d}\|^2$. ‡ h , º € • w R $r=\|\boldsymbol{p}\|=\frac{1}{\sqrt{6}}$, $\|\boldsymbol{a}\|=\|\boldsymbol{b}\|=\|\boldsymbol{c}\|=\|\boldsymbol{d}\|=\sqrt{\frac{3}{2}}$ › b ; b ” .]

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[1] > * É , g ô , O ˜ Ì , • ó , Å ¢ E : ¶ , é , 2017.

[2] h æ c , Š ¦ , a ™ , ² i $ Ì , • ó , Å • ü u ü ¶ , é , 2017.

[3] > Û Ä , ¢ E : ¶ Ö ó , http://www7b.biglobe.ne.jp/~h-kuroda/pdf/text_linear_algebra.pdf, (2022 å 7 D € °)